Name: $\qquad$
Please show all work and justify your answers.

1. Expand decimal 69 in binary, octal and hexadecimal. Expand hex ACA in decimal.
2. Use Euclid's algorithm to find $\operatorname{gcd}(77,76)$ and the Bézout coefficients.
3. Solve the system of congruences: $x \equiv 2 \bmod 4,2 x \equiv 3 \bmod 13, x \equiv 1 \bmod 5$.
4. For which $n>0$ is $n^{n}>(n+1)$ !? Prove your assertion.
5. (a) What augmented matrix $A$ represents the linear system $3 x+3 y+2=0, z=2 y+2 x$ ? Use Gauss-Jordan elimination to find the reduced row echelon form of $A$. Show steps.
(b) Use (a) to find all solutions to the system in terms of the free variable(s).
6. Let $A=\left[\begin{array}{rrr}-1 & 1 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1\end{array}\right], B=\left[\begin{array}{lll}0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{array}\right]$.
(a) Compute the determinant of $A$. Show steps. What does your answer say about the effect on volumes by the linear transformation of Euclidean space given by $x \mapsto A x$ ?
(b) Compute $A^{-1}$ (show steps) and use (a) to solve the matrix system $A X=B$ for $X$.
7. (a) Find the characteristic polynomial of $A=\left[\begin{array}{rr}-4 & 13 \\ 7 & 2\end{array}\right]$ and the eigenvalues of $A$.
(b) For each of the eigenvalues you found in (a) find corresponding eigenvectors.
(c) Find an invertible matrix $P$ such that $P^{-1} A P$ is diagonal.
(d) Sketch the eigenspaces. Describe geometrically the plane transformation $x \mapsto A x$.
8. How many ways can the 9 Supreme Court justices seat themselves behind a round table so that the 6 men sit with at most one male neighbor?
9. Suppose $45 \%$ of email you receive is not spam, $2 \%$ of legitimate email contains the words "hair augmentation" and $14 \%$ of spam messages contain those words. If you receive a message containing those words, what's the probability that it's spam?
10. Express the probability of being within $\pm 4$ of the mean in terms of the error function, if the probability density is normal with mean -1 and standard deviation 5 .

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | total (100) |
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