Name: _

Please show all work and justify your answers.

- 1. Expand decimal 69 in binary, octal and hexadecimal. Expand hex ACA in decimal.
- 2. Use Euclid's algorithm to find gcd(77, 76) and the Bézout coefficients.
- 3. Solve the system of congruences: $x \equiv 2 \mod 4, 2x \equiv 3 \mod 13, x \equiv 1 \mod 5$.
- 4. For which n > 0 is $n^n > (n+1)!$? Prove your assertion.
- 5. (a) What augmented matrix A represents the linear system 3x+3y+2=0, z=2y+2x? Use Gauss-Jordan elimination to find the reduced row echelon form of A. Show steps.
 - (b) Use (a) to find all solutions to the system in terms of the free variable(s).

6. Let
$$A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$
, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

- (a) Compute the determinant of A. Show steps. What does your answer say about the effect on volumes by the linear transformation of Euclidean space given by $x \mapsto Ax$?
- (b) Compute A^{-1} (show steps) and use (a) to solve the matrix system AX = B for X.
- 7. (a) Find the characteristic polynomial of $A = \begin{bmatrix} -4 & 13 \\ 7 & 2 \end{bmatrix}$ and the eigenvalues of A.
 - (b) For each of the eigenvalues you found in (a) find corresponding eigenvectors.
 - (c) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.
 - (d) Sketch the eigenspaces. Describe geometrically the plane transformation $x \mapsto Ax$.
- 8. How many ways can the 9 Supreme Court justices seat themselves behind a round table so that the 6 men sit with at most one male neighbor?
- 9. Suppose 45% of email you receive is not spam, 2% of legitimate email contains the words "hair augmentation" and 14% of spam messages contain those words. If you receive a message containing those words, what's the probability that it's spam?
- 10. Express the probability of being within ± 4 of the mean in terms of the error function, if the probability density is normal with mean -1 and standard deviation 5.

1	2	3	4	5	6	7	8	9	10	total (100)