ADVANCED EXAMINATION ◻ TOPOLOGY ◻ February 27, 1998

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Name: ______________________________

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Work any eight out of 10 problems. Please, indicate which problems you are doing in the top parts of the boxes above. Show all work.

1. Suppose X is a topological space, I is a set, and $\{A_i \colon i \in I\}$ is a collection of nonempty connected subsets of X. Furthermore, suppose that $A_i \cap A_j \neq \emptyset$ for all $i, j \in I$. Prove that $\bigcup_{i \in I} A_i$ is connected.

2. Suppose X is an infinite set and let $\mathcal{T} = \{U \subseteq X \colon X \setminus U \text{ is finite}\} \cup \{\emptyset\}$.
   (a) Prove that $\mathcal{T}$ is a topology.
   (b) Prove that the topological space $(X, \mathcal{T})$ is T_1, but not T_2.
   (c) Prove that $(X, \mathcal{T})$ is connected.

3. Suppose I is a set and $\{X_i \colon i \in I\}$ is a collection of topological spaces. Suppose that for each $i \in I$ we have a closed subset $A_i \subseteq X_i$. Prove that $\prod_{i \in I} A_i$ is closed subset of $\prod_{i \in I} X_i$ with the product topology.

4. Suppose X and Y are topological spaces, X is compact and $f : X \to Y$ is a continuous surjection. Prove that Y is compact.

5. Prove that an open connected subset of $\mathbf{R}^n$ is path connected.

6. Suppose X and Y are connected topological spaces. Prove that $X \times Y$ is connected. You may use the result of problem 1.

7. Suppose X is a contractible topological space. Prove that X is path connected.

8. Suppose U is a convex subset of $\mathbf{R}^n$. Prove that U is contractible.

9. Prove that S^n — the unit sphere in $\mathbf{R}^{n+1}$ is homotopy equivalent to $\mathbf{R}^{n+1} \setminus 0$.

10. Suppose X is a path connected topological space. Prove that $H_0(X; \mathbf{Z}) \cong \mathbf{Z}$.