ADVANCED EXAMINATION □ COMPLEX ANALYSIS □ April 4, 1998

Dr. Dmitry Gokhman / Division of Mathematics and Statistics / UT San Antonio

Name: ____

#	#	#	#	#	#	#	#	total (160)

Work any 8 problems. Indicate which problems you are doing in the top parts of the boxes above. Unless otherwise indicated, assume that D is a domain in $\mathbf{C}$ and Σ is the Riemann sphere.

1. Prove that if f is holomorphic on D and has constant modulus, then f is constant.

2. Suppose f_n $(n \in \mathbf{Z}^+)$ are holomorphic functions on D which sum uniformly to $f = \sum_{n=1}^{\infty} f_n$. Prove that f is holomorphic on D.

3. Find the Maclaurin series expansion of $\dfrac{1}{(1+z)^2}$. Determine the radius of convergence r. Prove that the series converges uniformly on compact subsets of the open disk $\{z\colon\ |z| < r\}$.

4. State and prove the Maximum Modulus Principle. You may use one of the following (a) Cauchy's Integral Formula, (b) the Open Mapping Theorem.

5. State and prove Liouville's theorem. You may use Cauchy's Integral Formula.

6. (a) Evaluate $\displaystyle\int \frac{dz}{(z^2 - 4iz - 3)^3}$ around the circle $\{z\colon\ |z| = 2\}$.

   (b) Evaluate $\displaystyle\int \frac{dz}{\sin z}$ around the unit circle.

7. Find the Laurent series for $\dfrac{1}{z^2 - 4iz - 3}$ valid in the annulus $\{z\colon 1 < |z| < 3\}$.

8. Suppose $f(z) = \displaystyle\sum_{k=-\infty}^{\infty} a_k z^k$ near 0. Prove that the residue of $f(z)$ at 0 is a_{-1}.

9. Suppose $w_1,\ w_2,\ w_3$ are distinct points of $\mathbf{C}$. Prove that there exists a unique Möbius transformation $T\colon \Sigma \to \Sigma$ such that $T(0) = w_1$, $T(1) = w_2$, and $T(\infty) = w_3$.

10. Suppose $f\colon \Sigma \to \Sigma$ is meromorphic. Prove that

    (a) The number of poles of f is finite.

    (b) f is a rational function.