

Linear algebra glossary

Vector space: A vector space V is a nonempty set with addition and scalar multiplication.

The operations must satisfy certain axioms for *arbitrary* vectors u, v, w in V and numbers a, b .

Closure: $u + v$ and au are in V

Commutative: $u + v = v + u$

Associative: $u + (v + w) = (u + v) + w$ and $(ab)u = a(bu)$

Zero: there exists a zero vector 0 such that $u + 0 = u$

Additive inverse: there exists $-u$ such that $u + (-u) = 0$

Distributive: $a(u + v) = au + av$ and $(a + b)u = au + bu$

Unitary: $1u = u$

Superposition principle I: If v_1, \dots, v_n are vectors in V , then any linear combination $c_1v_1 + \dots + c_nv_n$ is in V .

Subspace: A subspace of V is a nonempty subset H of V that is closed under addition and scalar multiplication.

Span: If S is a subset of V , the span of S is the set (in fact, subspace) of all linear combinations of vectors in S .

Linear map: A linear map is a function between vector spaces $T: V \rightarrow W$ that preserves addition and scalar multiplication. Specifically $T(u + v) = T(u) + T(v)$ and $T(au) = aT(u)$.

Superposition principle II: If T is linear, then $T(c_1v_1 + \dots + c_nv_n) = c_1T(v_1) + \dots + c_nT(v_n)$

Linear independence: A sequence of vectors v_1, \dots, v_n is linearly independent means that the homogeneous vector equation $c_1v_1 + \dots + c_nv_n = 0$ has only the trivial solution $c_1 = \dots = c_n = 0$.

Basis: A basis for V is a linearly independent sequence of vectors v_1, \dots, v_n that spans V .

Dimension: Dimension of V is the (unique) number of elements in any basis for V .

Coordinates: If v is a vector in V and $\mathcal{B} = \{b_1, \dots, b_n\}$ is a basis for V , the coordinate vector $[v]_{\mathcal{B}}$ consists of the coefficients (weights) c_1, \dots, c_n of the unique expansion $v = c_1b_1 + \dots + c_nb_n$.