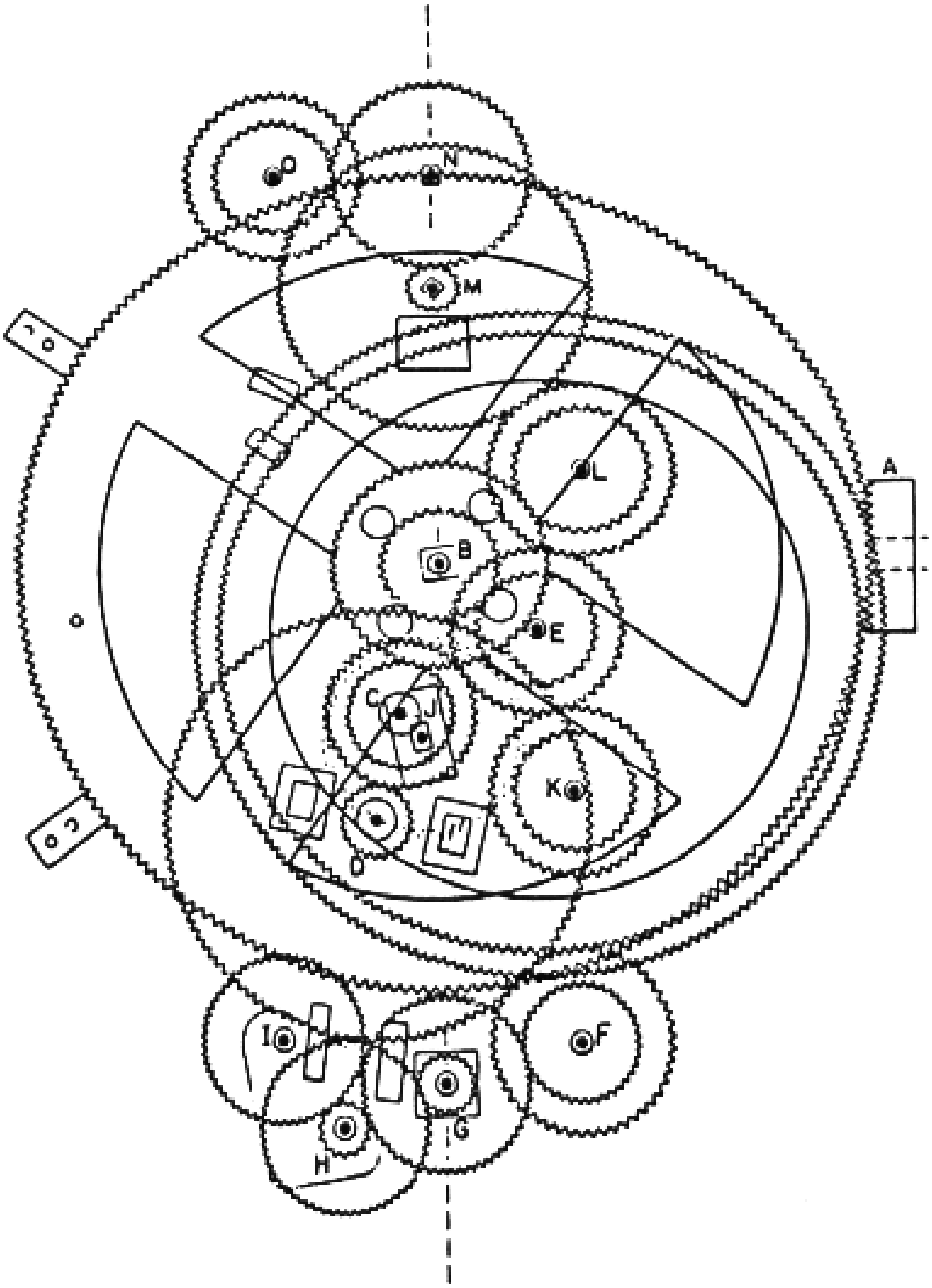

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GEARS FROM THE ANCIENT GREEKS

Professor Sir Christopher Zeeman

THE ANTIKYTHERA MECHANISM IS

- The most sophisticated scientific instrument surviving from antiquity
- An astronomical calculator with precision gearing
- Contains 32 bronze gears
- Contains a differential gear
- Accurate to 1 part in 40,000
- Has forced the historians of science to reassess the high technology of the ancient Greeks.

(• There is only one other surviving example of precision gearing made before 1300 AD, and that is much more primitive).

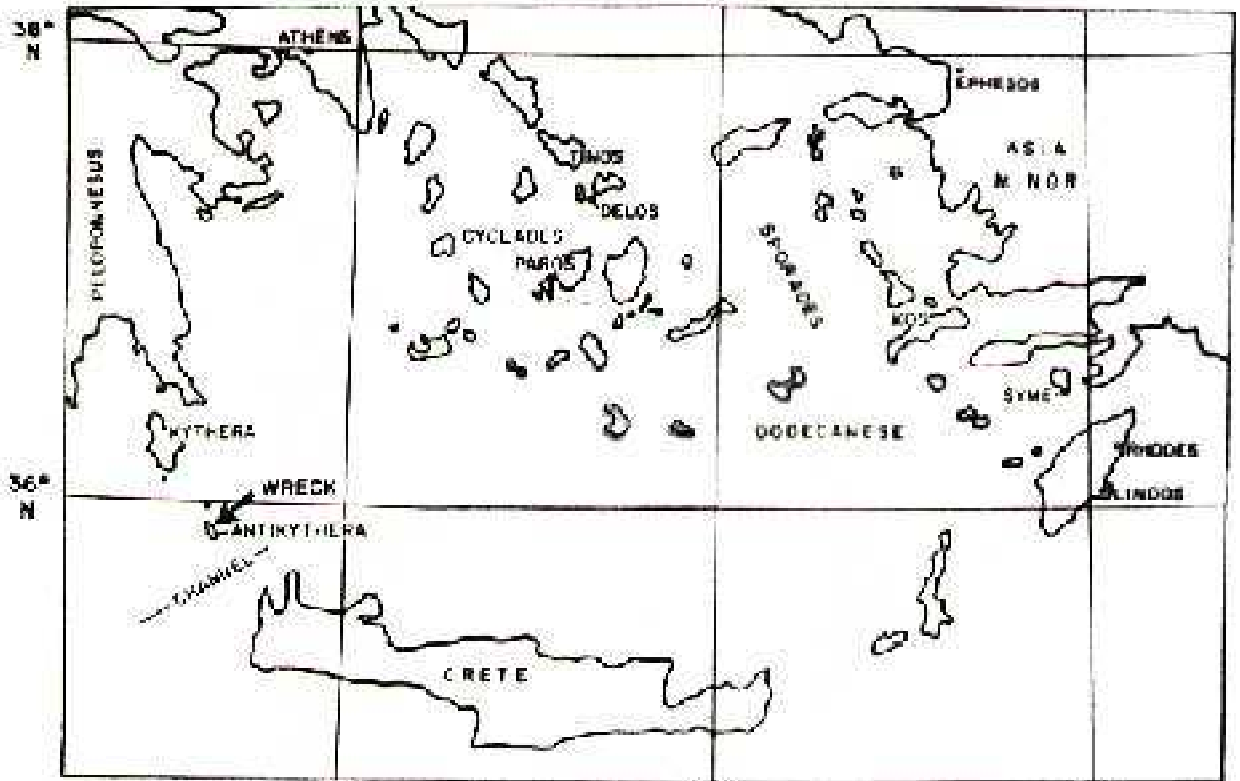


Fig. 1. Map of the Aegean Islands.

The Antikythera Mechanism was

87 BC made in Rhodes.

70 BC shipwrecked off Antikythera.

Calcified by 2000 years underwater.

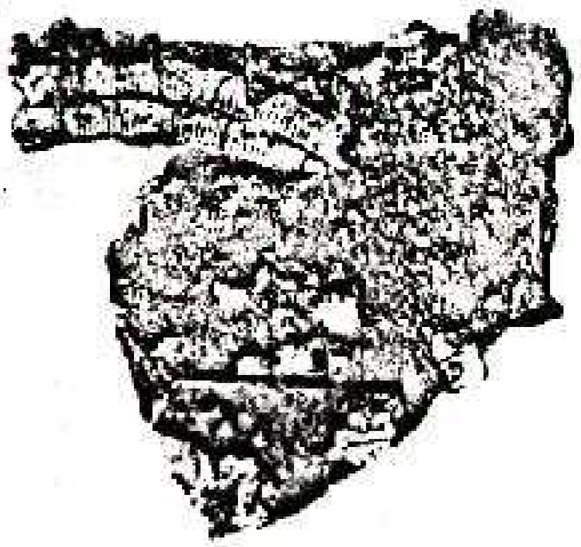
1900 AD discovered by sponge fishermen at depth 42 metres.

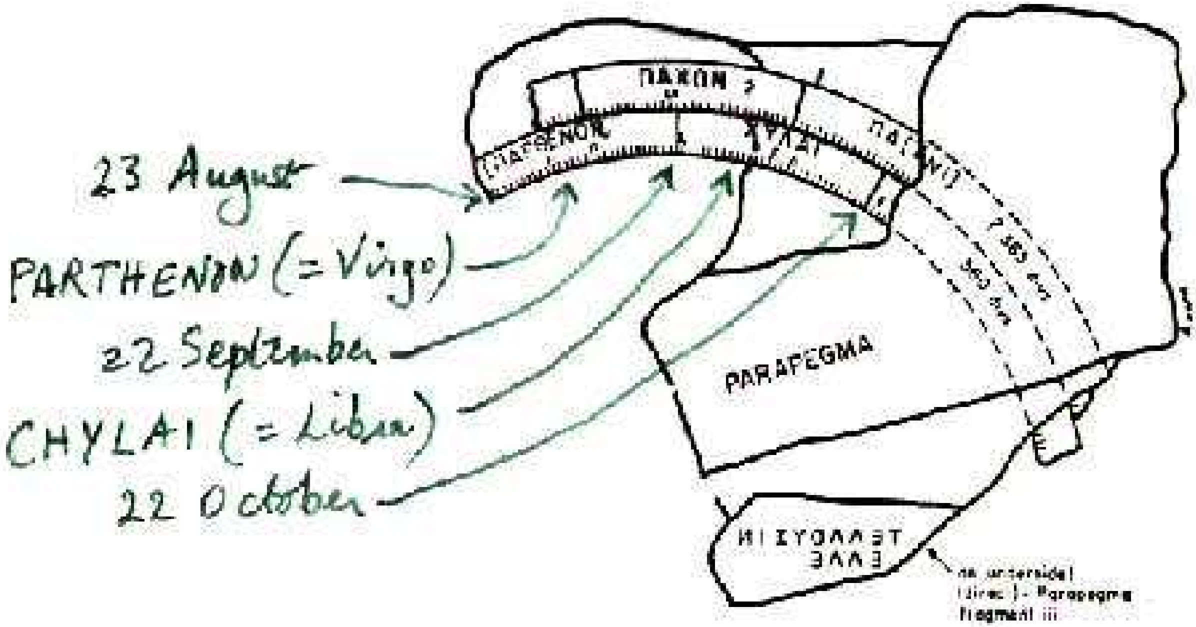
1902 dried out & split into 6 pieces, cleaned & displayed in the Athens Archaeological Museum.

1972 interior X-rayed by Price & Karakalos.

1974 deciphered by Derek de Solla Price.

1986 working model constructed by Alan Bromley.

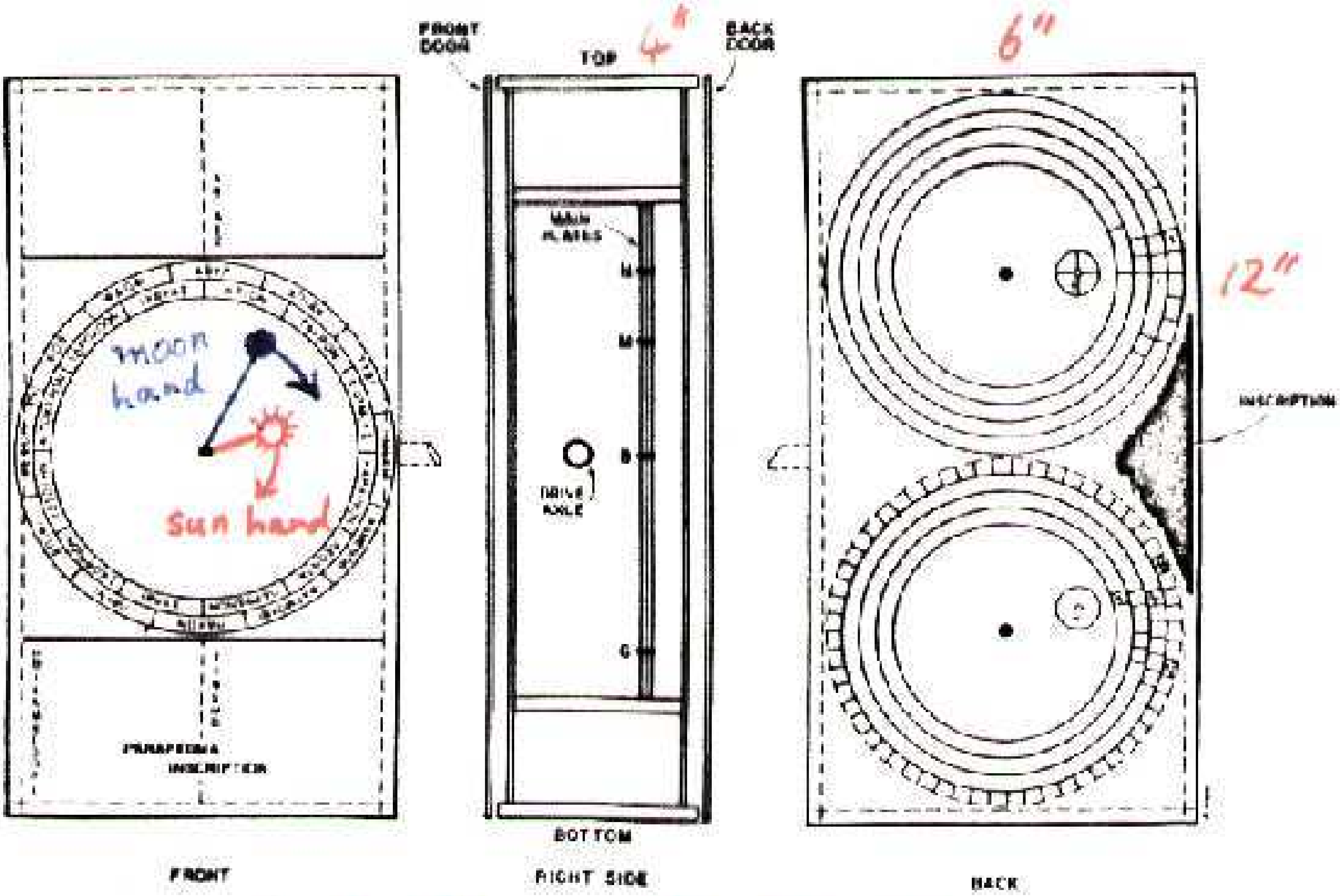




Front dial fragment.

Inner scale: 360° = 12 signs of the Zodiac

Outer scale: 365 days = 12 months of 30 days each
+ 5 epagomenal (holiday) days.



<u>Conjectured reconstruction of case.</u>

<u>Front Face</u>

2 hands showing positions of sun & moon

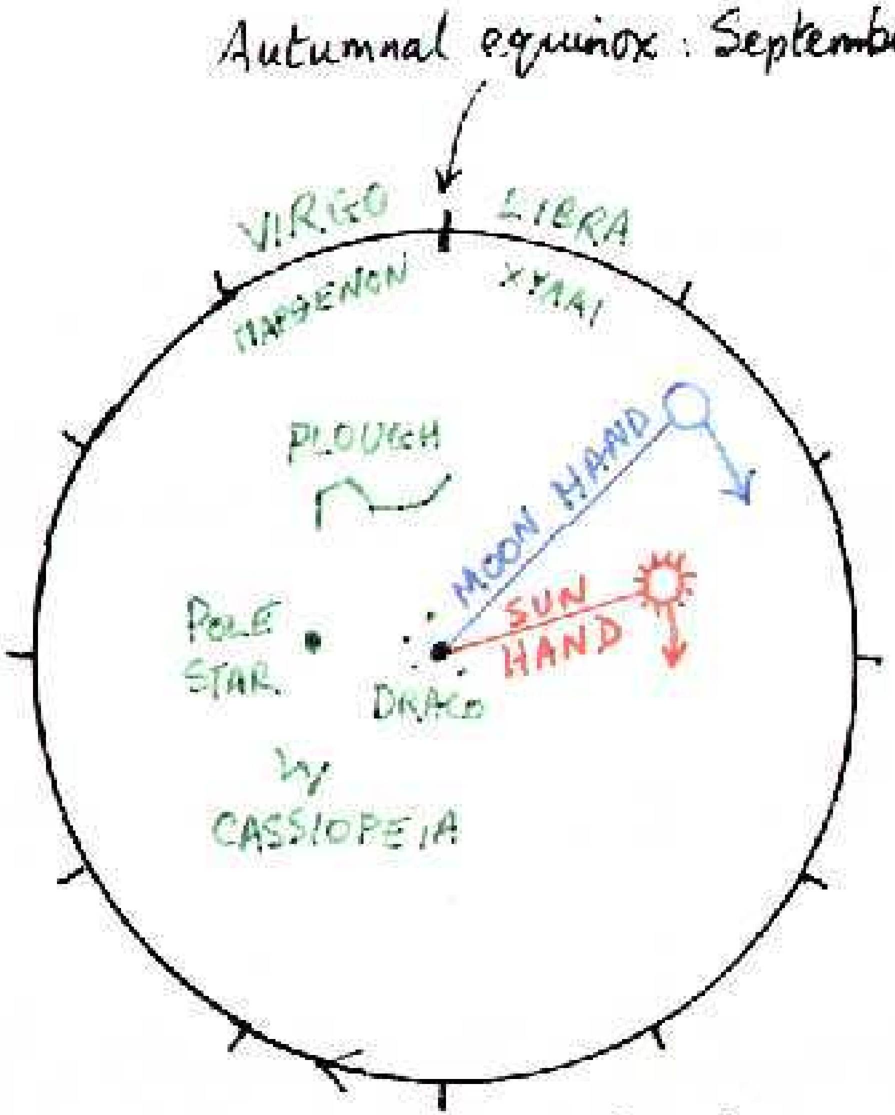
<u>Back Face</u>

<u>Top</u> Eclipses.

<u>Bottom</u> Phases of the moon

Question: Why do the hands go clockwise?

Answer: Because its a picture of the Northern sky.



ECLIPTIC = path of the sun against the fixed stars, divided into 12 constellations of the Zodiac.

Question: What is the ratio of speeds of the two hands?

Answer. The number of months in a year:

Question. What is a month?

Answer:

A **synodic month** = period between two new moons
(ordinary people)
$\simeq 29\frac{1}{2}$ days.

Synodic (Greek: συν + οδος = meeting)
= conjunction of sun & moon.

A **sidereal month** = period of the moon's orbit round the ecliptic
(astronomers)
$\simeq 27\frac{1}{3}$ days

Sidereal (Latin: sidereus: stars) — of the stars.

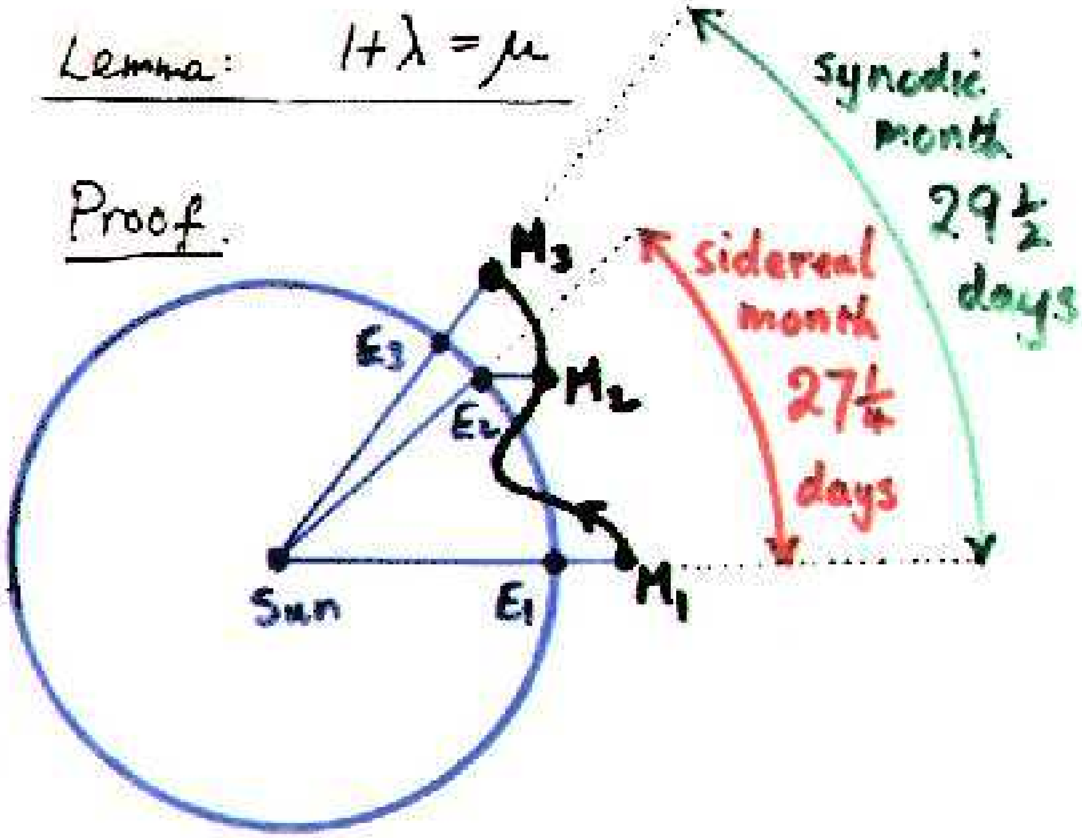
Let λ = no. synodic months/year, $= 12.3687\ldots$

μ = no. sidereal months/year $= 13.3687\ldots$

Question: Why do these differ by exactly 1?

Lemma: $1+\lambda=\mu$

Proof.



Seen from the earth, let

a = angular velocity of the moon M (large)

b = " " " " sun S (small)

c = " " " M relative to S. (large)

Then $c = a - b$

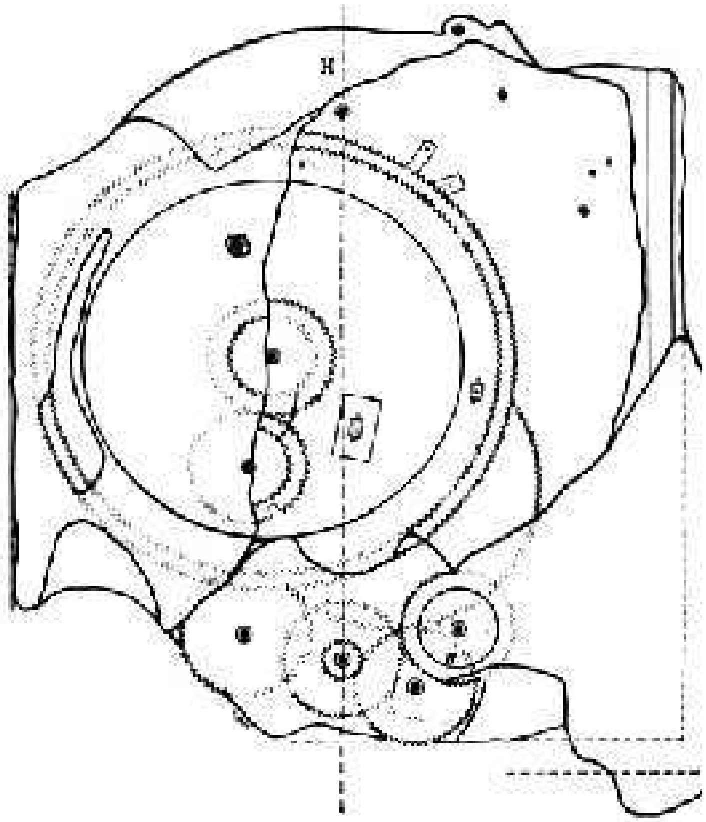
$\therefore\ b + c = a$

$\therefore\ 1 + \frac{c}{b} = \frac{a}{b}$

λ = number of synodic months per year.

μ = number of sidereal months per year

$\therefore\ 1+\lambda = \mu.$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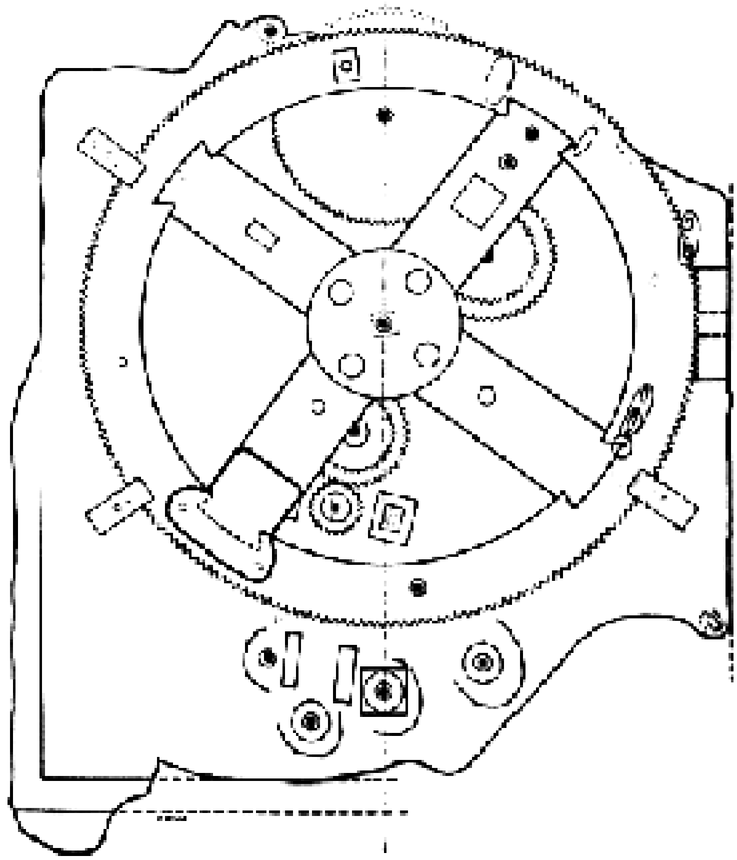
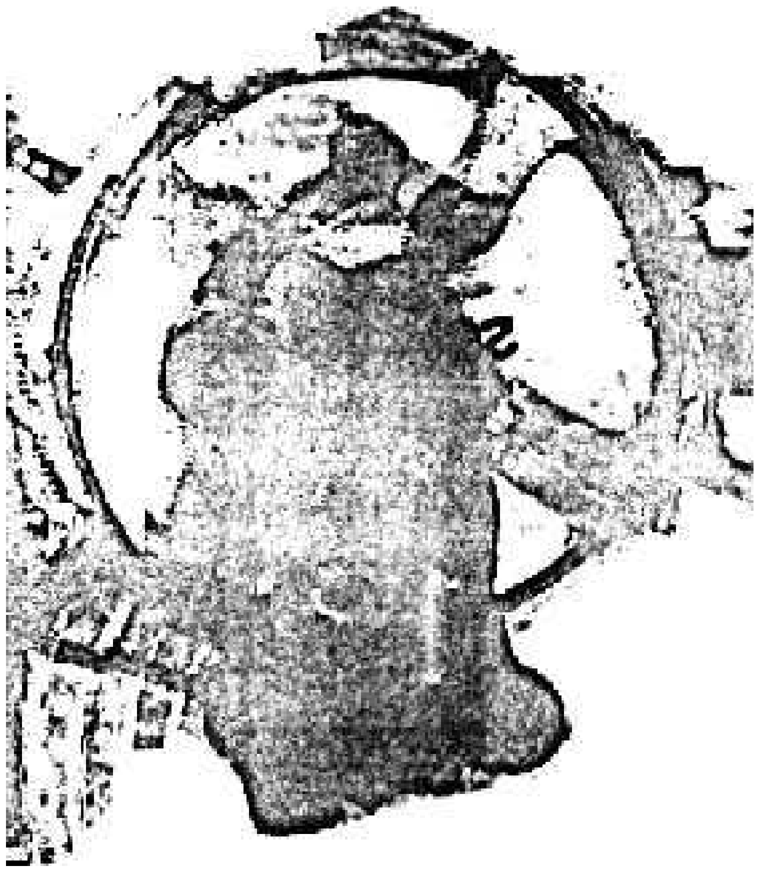
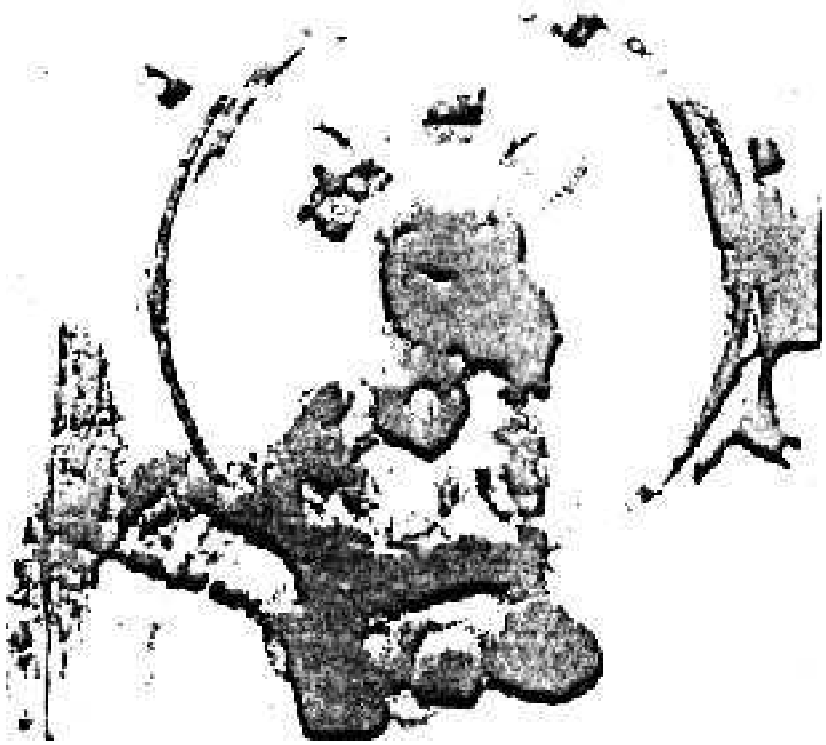
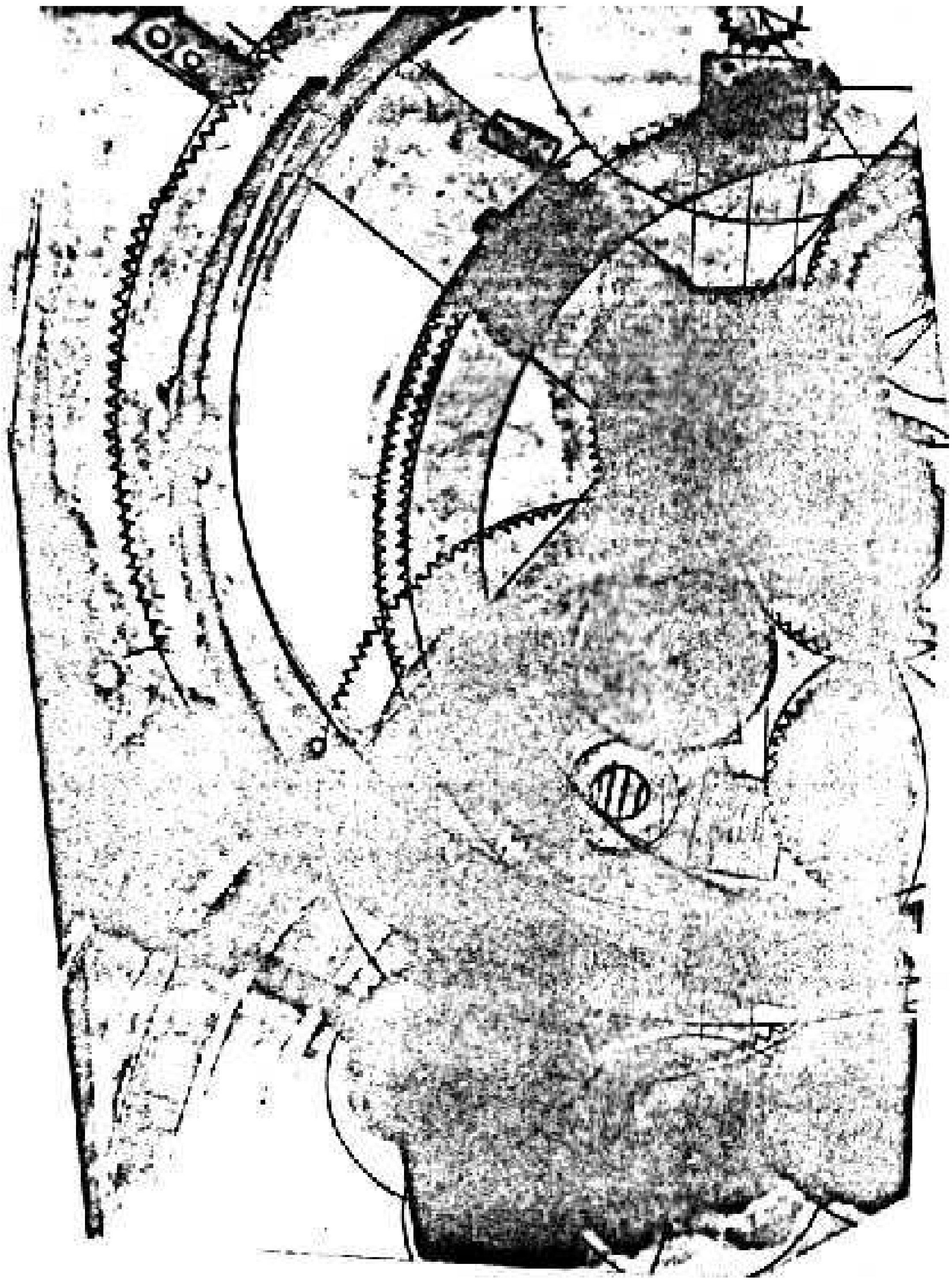
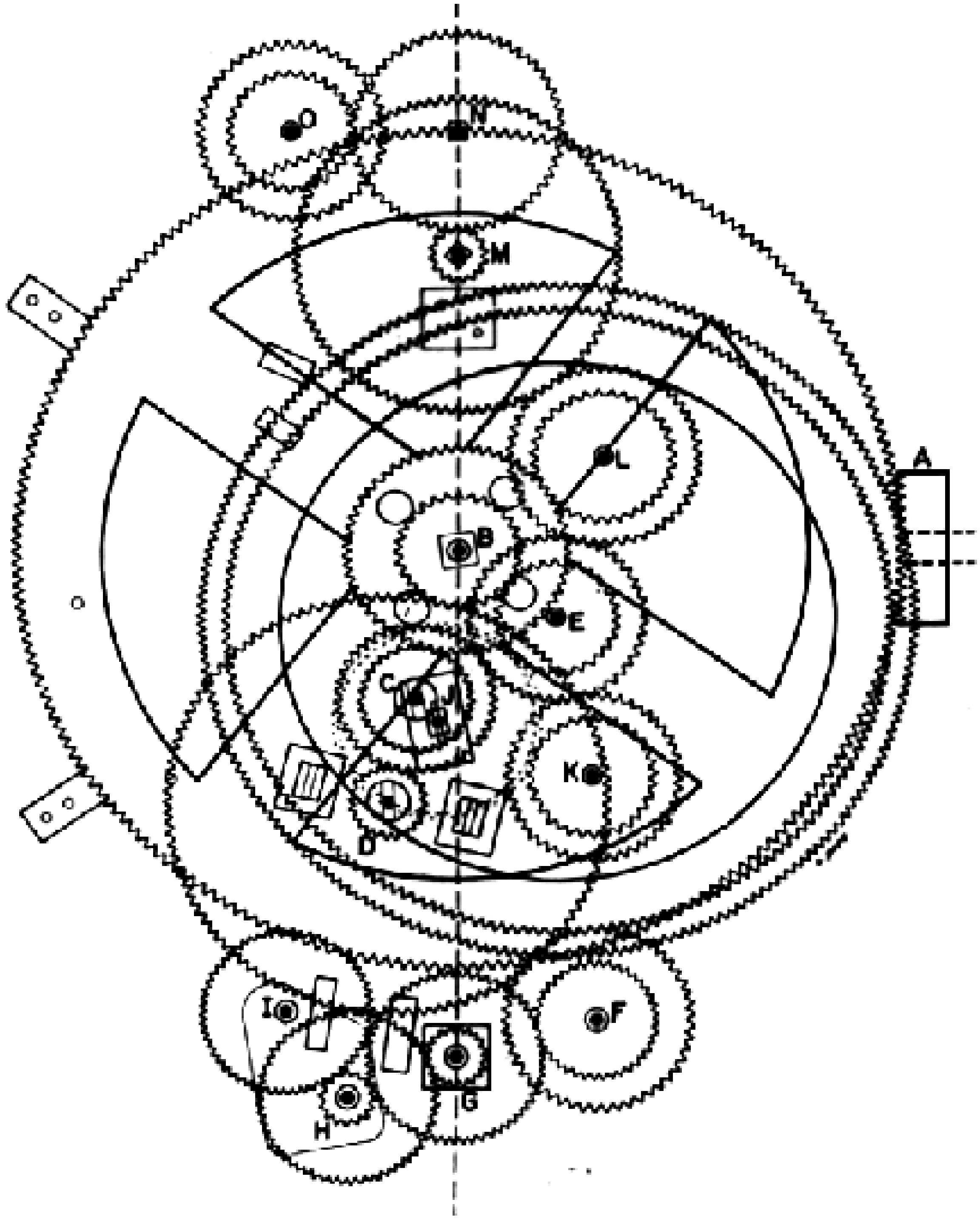


1972 Professor Derek de Solla Price (Yale, historian of science) & Dr. Chas. Karakalos (Athens, physicist) took X-rays of the interiors of the lumps of calcified metal, & found traces of all the gear wheels, and counted the teeth.









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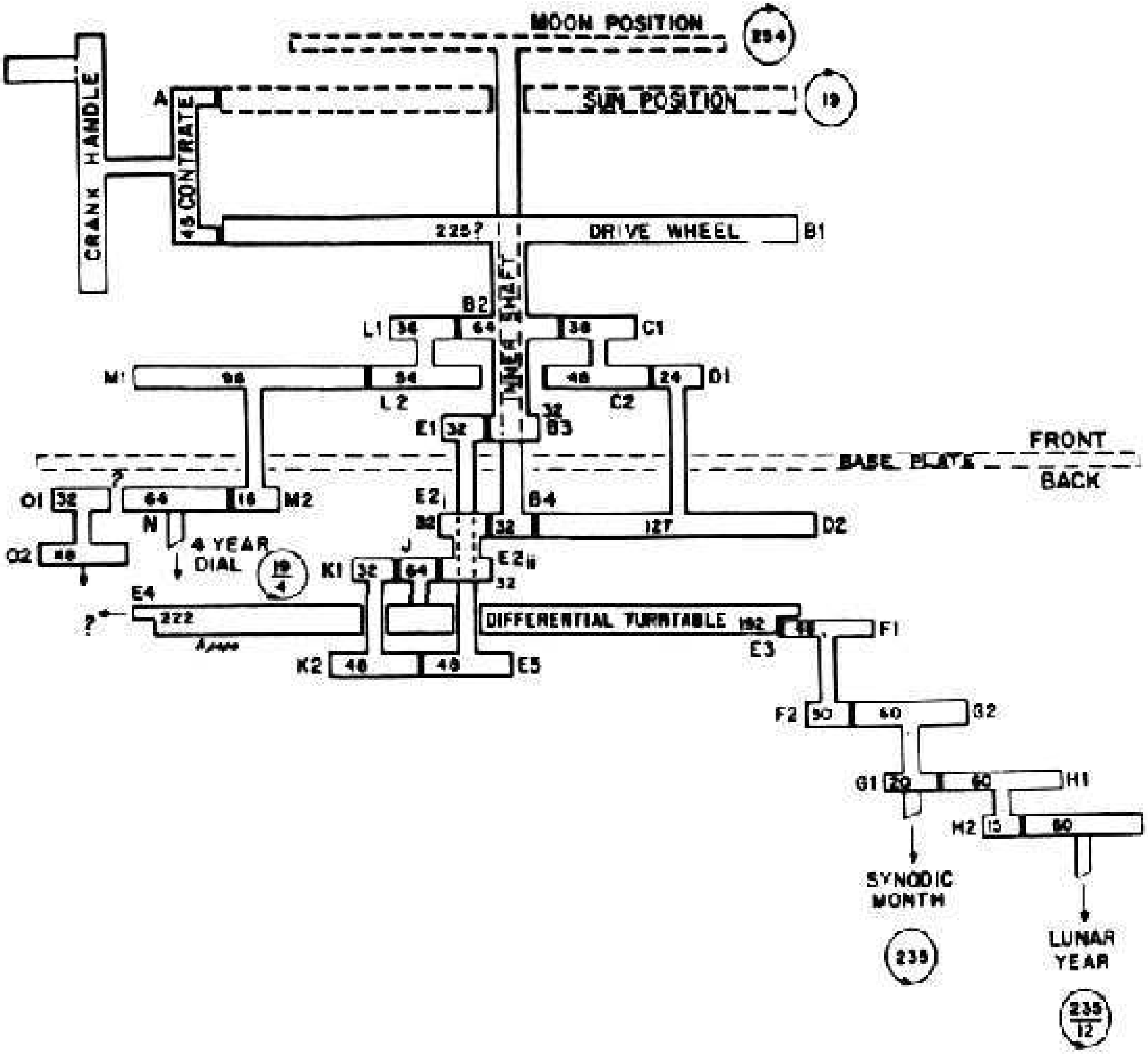
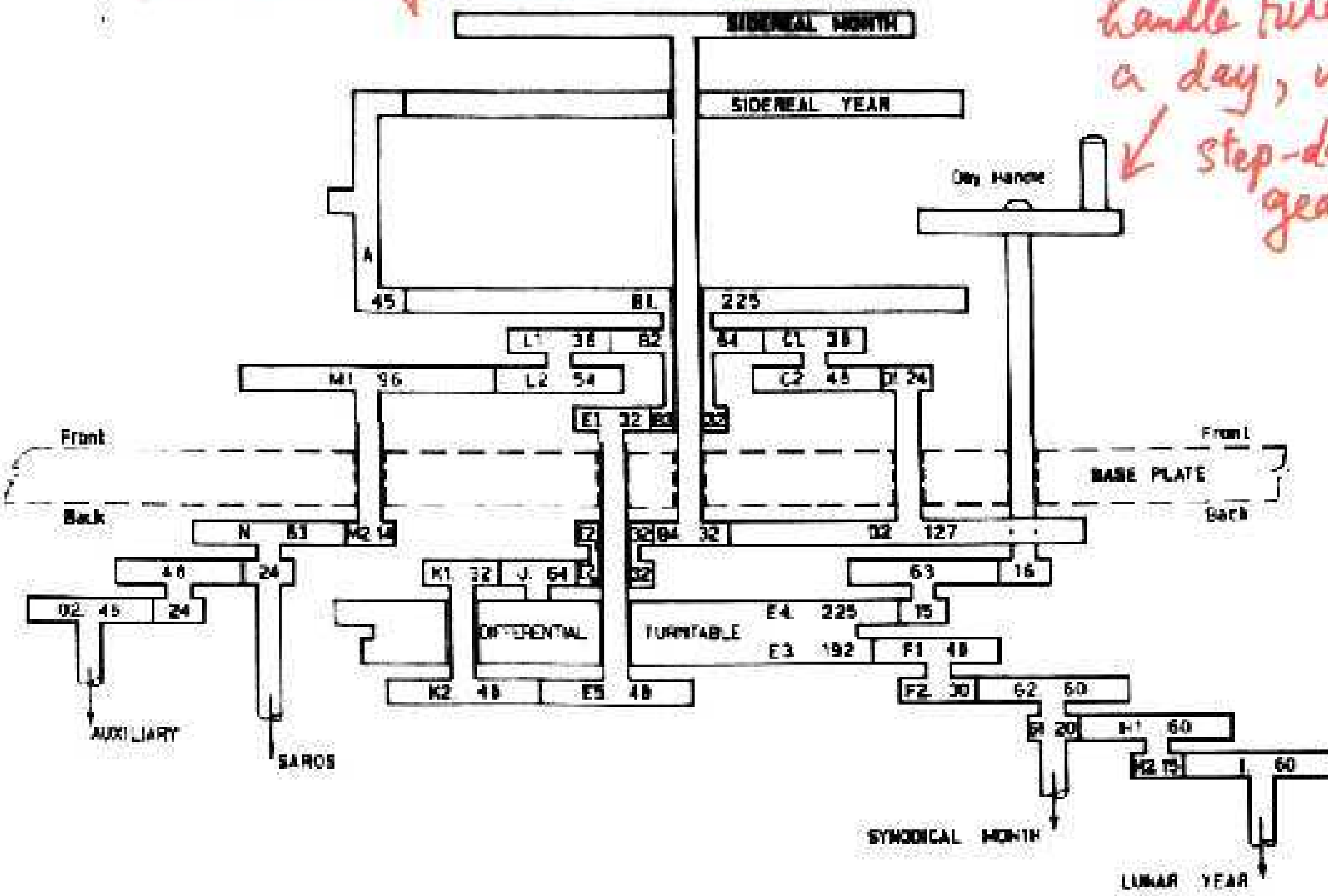
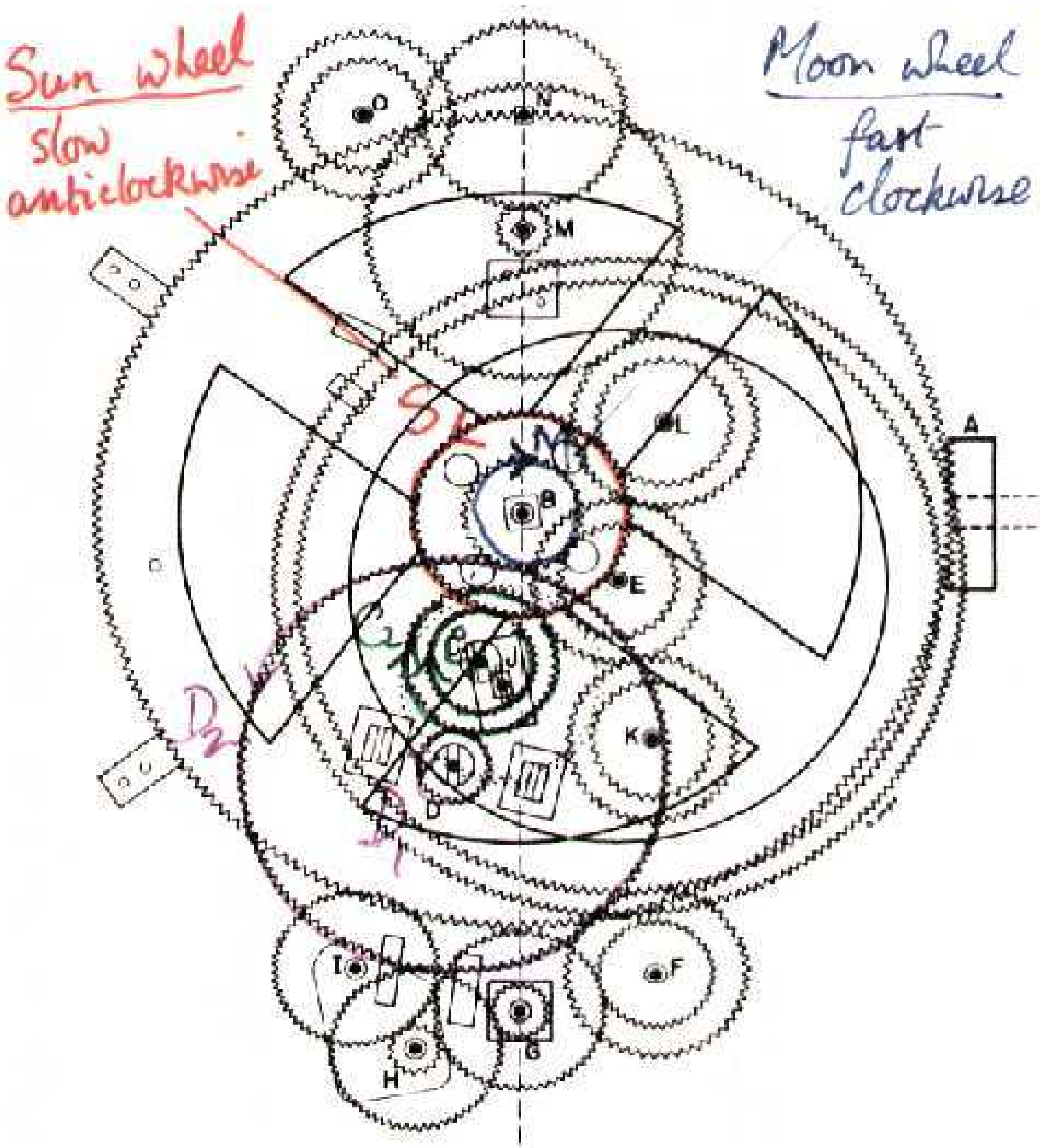


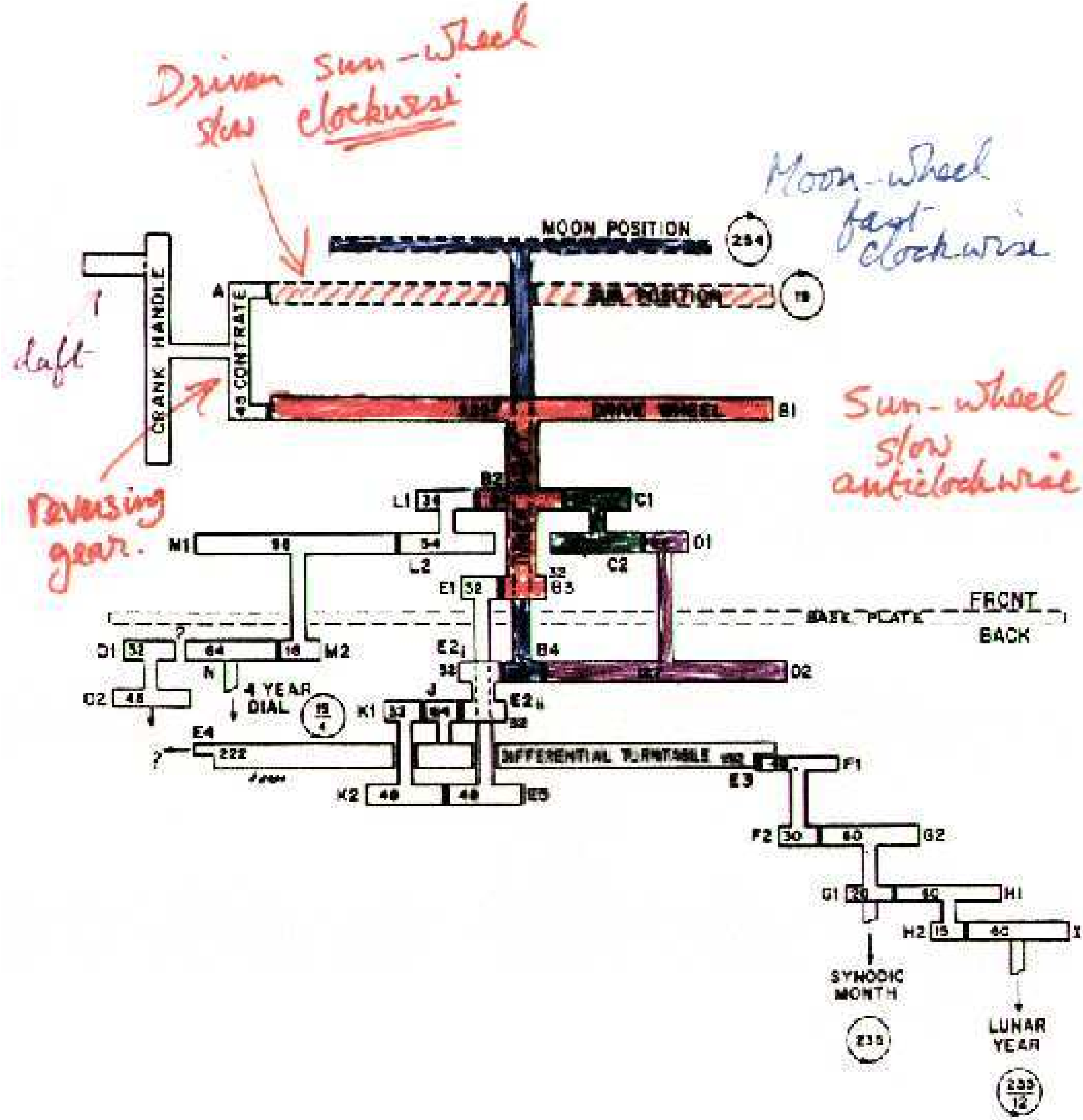
MOON POSITION
254
SUN POSITION
19
CRANK HANDLE
A
48 CONTRATE
225?
DRIVE WHEEL
B1
INNER SHAFT
B2
L1
36
64
38
C1
M1
96
54
48
24
D1
L2
C2
32
E1
32
B3
FRONT
BASE PLATE
BACK
O1
32
?
64
16
M2
N
O2
48
4 YEAR
DIAL
19
4
E2
B2
B4
32
127
D2
K1
32
J
64
E2
32
E4
222
?
DIFFERENTIAL TURNTABLE
192
E3
F1
K2
48
48
E5
F2
30
60
G2
G1
20
60
H1
H2
15
60
SYNODIC
MONTH
235
LUNAR
YEAR
235
12

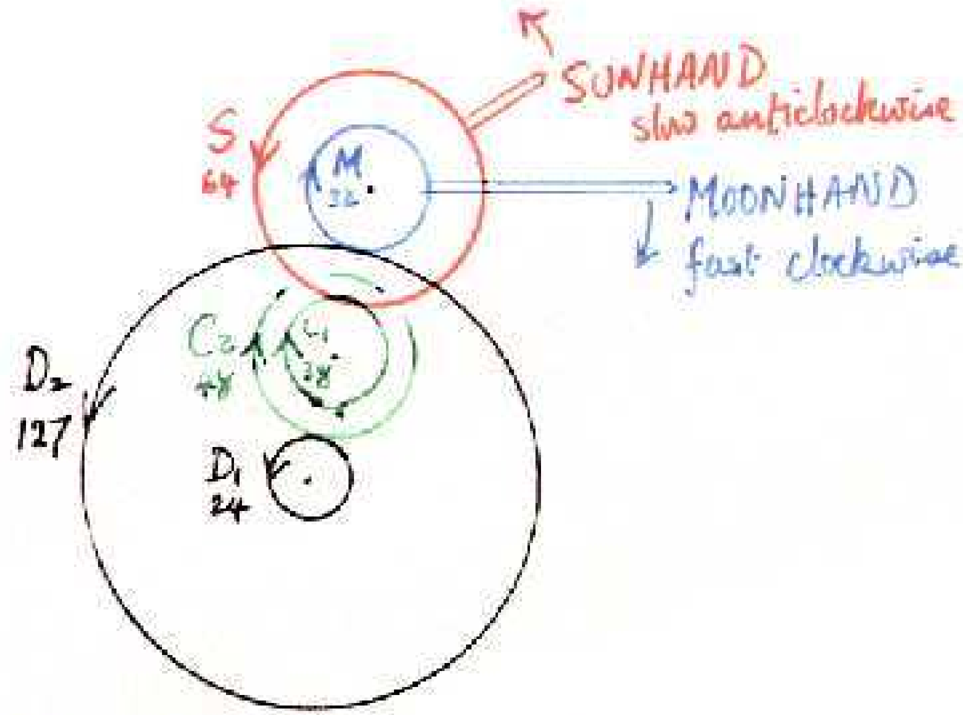
Allan Bromley's Improvement (1986)

The crank handle turns once a day, with a step-down gearing




Sun wheel
slow
anticlockwise
Moon wheel
fast
clockwise
O
N
M
L
A
B
E
K
I
F
G
H


Driven Sun-wheel slow clockwise
Moon-wheel fast clockwise
MOON POSITION
254
19
left
CRANK HANDLE
A
45 CONTRATE
DRIVE WHEEL
B1
Sun-wheel slow anticlockwise
Reversing gear.
B2
L1 36
C1
M1
96
54
L2
D1
C2
E1 32
32
B3
FRONT
BACK
D1 32
64
16 M2
N
C2 48
4 YEAR DIAL
19/4
E2
B4
D2
J
K1 32
E2
32
E4
222
DIFFERENTIAL TURNTABLE
E3
F1
K2 48
48
E5
F2 30
60
G2
G1
60
H1
H2 15
60
I
SYNODIC MONTH
235
LUNAR YEAR
235/12



Moonwheel M drives D_2 fixed to D_1

D_1 drives C_2 fixed to C_1

C_1 drives S sunwheel.

$$\text{Gear ratio} = \frac{S}{C_1} \times \frac{C_2}{D_1} \times \frac{D_2}{M}$$

$$= \frac{64}{38} \times \frac{48}{24} \times \frac{127}{32}$$

$$= \frac{254}{19}$$

$= 13{\cdot}3684\ldots$ (compare with modern figure $13{\cdot}3687\ldots$)

$= \mu$, correct to 1 in 40,000.

$\boxed{19 \text{ yrs} = 235 \text{ mo}}$ $\therefore \lambda = \mu - 1 = \frac{235}{19}$ (Meton, 11th June 431 BC)

How did the Greeks achieve such accuracy?

PROBLEM OF MEASURING λ ACCURATELY.

1. The accuracy of 1 part in 40,000 is equivalent to measuring the length of the month to the nearest minute (1 month $\simeq$ 42480 minutes).
2. You need to take the average of many readings to get the <u>mean</u> month, because the length of the month varies (due to elliptic orbits).
3. The Greeks had no instruments to measure time so precisely.
4. Its almost impossible to observe the phases of the moon with such precision.
5. The Greeks didn't have real numbers or decimals in which to express the result.
6. Nor did they have the technique of dividing two real numbers, to calculate the ratio.
7. The Metonic ratio $\frac{235}{19}$ is in fact the <u>most accurate</u> approximation to λ by any rational p/q with $q \leq 80$.
8. So how did they do it?

ALGORITHM FOR CALCULATING λ

Let S = the sequence of numbers of new moons each year for 50 years

$= 12, 13, 12, 12, 13, 12, 13, \ldots$

Define the derived sequence DS to be the numbers of steps from each 13 to the next.

$$DS = 3, 2, 3, 3, 3, 2, 3, \ldots$$

Repeat the process:

$$D^2S = 2, 1, 1, 2, 1, 2, \ldots$$

$$D^3S = 3, 2, 3, 2, 3, 2, 2, 3, \ldots$$

$$D^4S = 2, 2, 3, \ldots$$

Choose the lower number in each sequence and make a continued fraction:

$$\lambda = 12 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \frac{1}{2}}}}$$

$$= 12 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{2}{5}}} = 12 + \cfrac{1}{2 + \frac{5}{7}}$$

$$= 12 + \frac{7}{19} = \frac{228 + 7}{19} = \frac{235}{19}$$

= Metonic ratio.

Definition

An **n-sequence** is a sequence of n's and $n+1$'s.

Hypothesis

Suppose X, Y are two periodic events $\left(\text{eg } \begin{array}{l} X = \text{new year} \\ Y = \text{new moon} \end{array}\right)$

Let $\lambda = \dfrac{\text{period } X}{\text{period } Y}$

Suppose (for simplicity) that $\begin{cases} \lambda > 1 \\ \lambda \text{ is irrational} \\ \text{there are no coincidences} \end{cases}$

Let S = the sequence of numbers of Y's between successive X's.

Call S an **unfolding of λ.**

Theorem

For each $q \geq 0$, $D^q S$ is an n_q-sequence, for some n_q.

$$\lambda = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cdots}}$$

Remarks

1. Algorithm gives $\begin{pmatrix}\text{metric} \\ \text{information}\end{pmatrix}$ from $\begin{pmatrix}\text{order} \\ \text{information}\end{pmatrix}$

   easy to use

   hard to measure but easy to calculate.

   easy to observe

2. $\exists$ uncountably many different unfoldings of λ.
3. Self-correcting.

Given S is an unfolding of λ.

Write $\lambda = n + r$

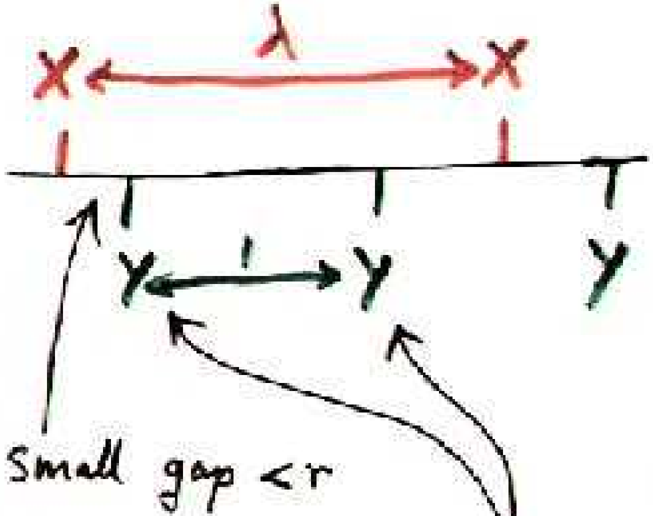
integer remainder, $0<r<1$

($r \neq 0$ because λ irrational)

Lemma 1 S is an n-sequence

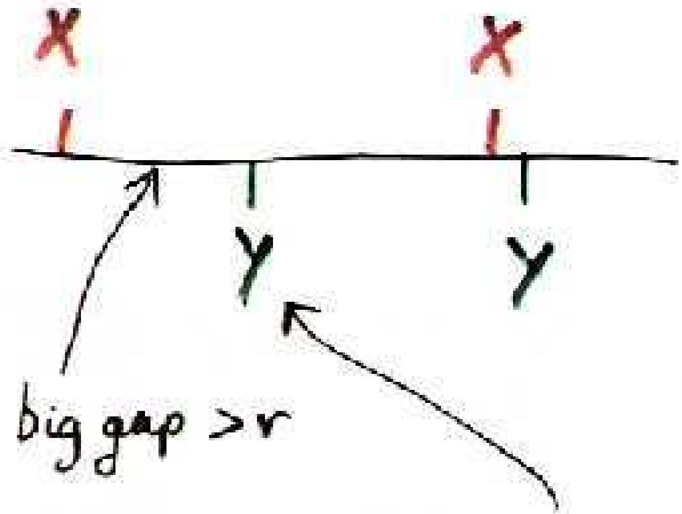
Proof. There are two types of "year":

<u>Fat year</u>



∴ room for $n+1$ Y's

<u>Thin year</u>



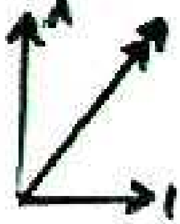
∴ only room for n Y's

Therefore in any year there are either n or $n+1$ "new moons".

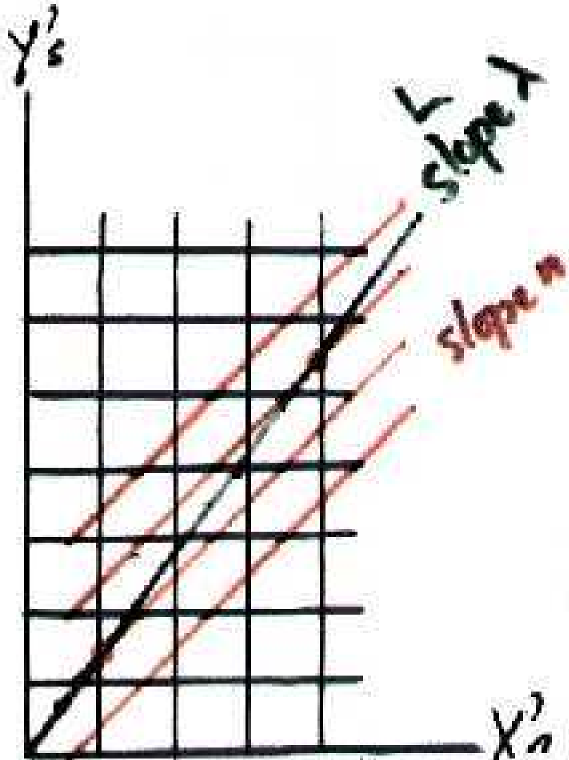
Proof of Lemma 2

Represent X's & Y's along the axes at unit intervals.

Velocity



(Unit of time = period of X)



S is represented by a line L of slope λ.

Let $\lambda = n + r$.

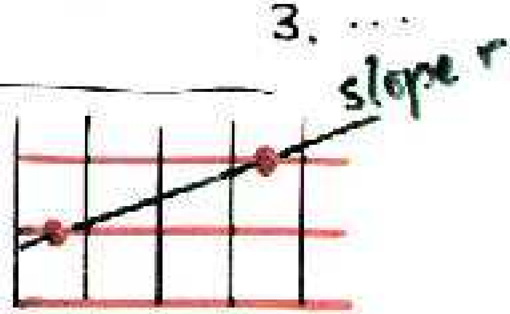
Mark in red lines of slope n.

x y y y x y x y x y y x ...

Read off the unfolding S: 2 1 1 2 ...

∴ derived sequence DS: 3, ...

Make the red lines horizontal by a sheer map

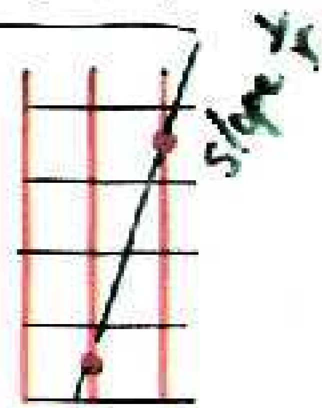


L now has slope $= \lambda - n = r$

Then flip over

L now has slope $\frac{1}{r} = D\lambda$

∴ DS is an unfolding of Dλ.



DS: 3 ....

Corollary 1 The derived sequence DS exists

Definition Define the <u>derived number</u> $D\lambda = \frac{1}{r}$

Lemma 2 DS is an unfolding of $D\lambda$

Corollary 2 $D^q S$ is an unfolding of $D^q \lambda$

Corollary 3 $D^q S$ is an n_q-sequence, for some n_q

Proof of the Theorem $\lambda = n_0 + r_0$

$$D\lambda = \frac{1}{r_0} = n_1 + r_1$$

$$D^2\lambda = \frac{1}{r_1} = n_2 + r_2, \quad \text{and so on.}$$

Therefore $\lambda = n_0 + \dfrac{1}{n_1 + r_1} = n_0 + \dfrac{1}{n_1 + \dfrac{1}{n_2 + r_2}}$

$$= n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cdots}}$$

Angular velocity of moonwheel = a

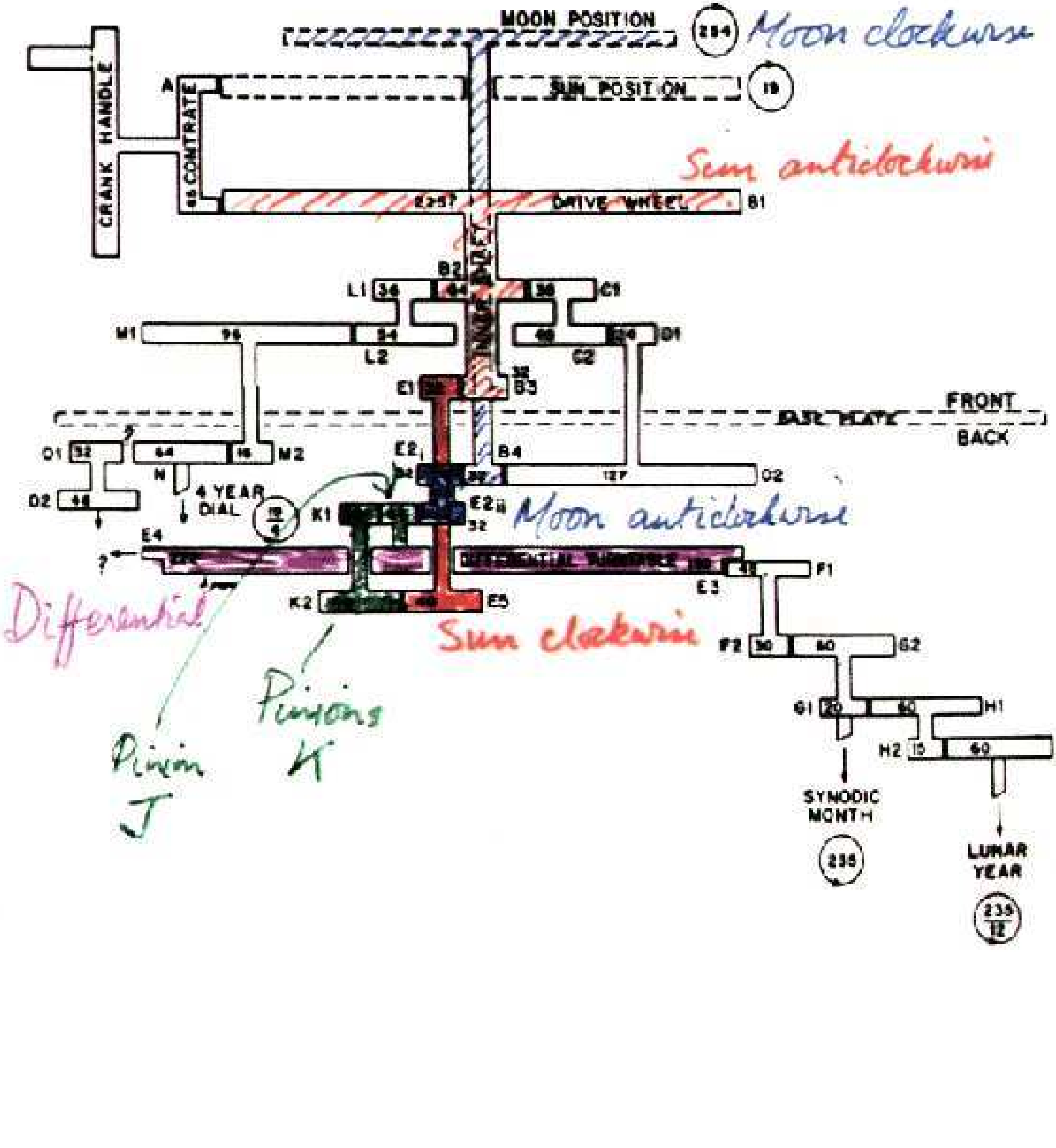
" " " Sunwheel = b

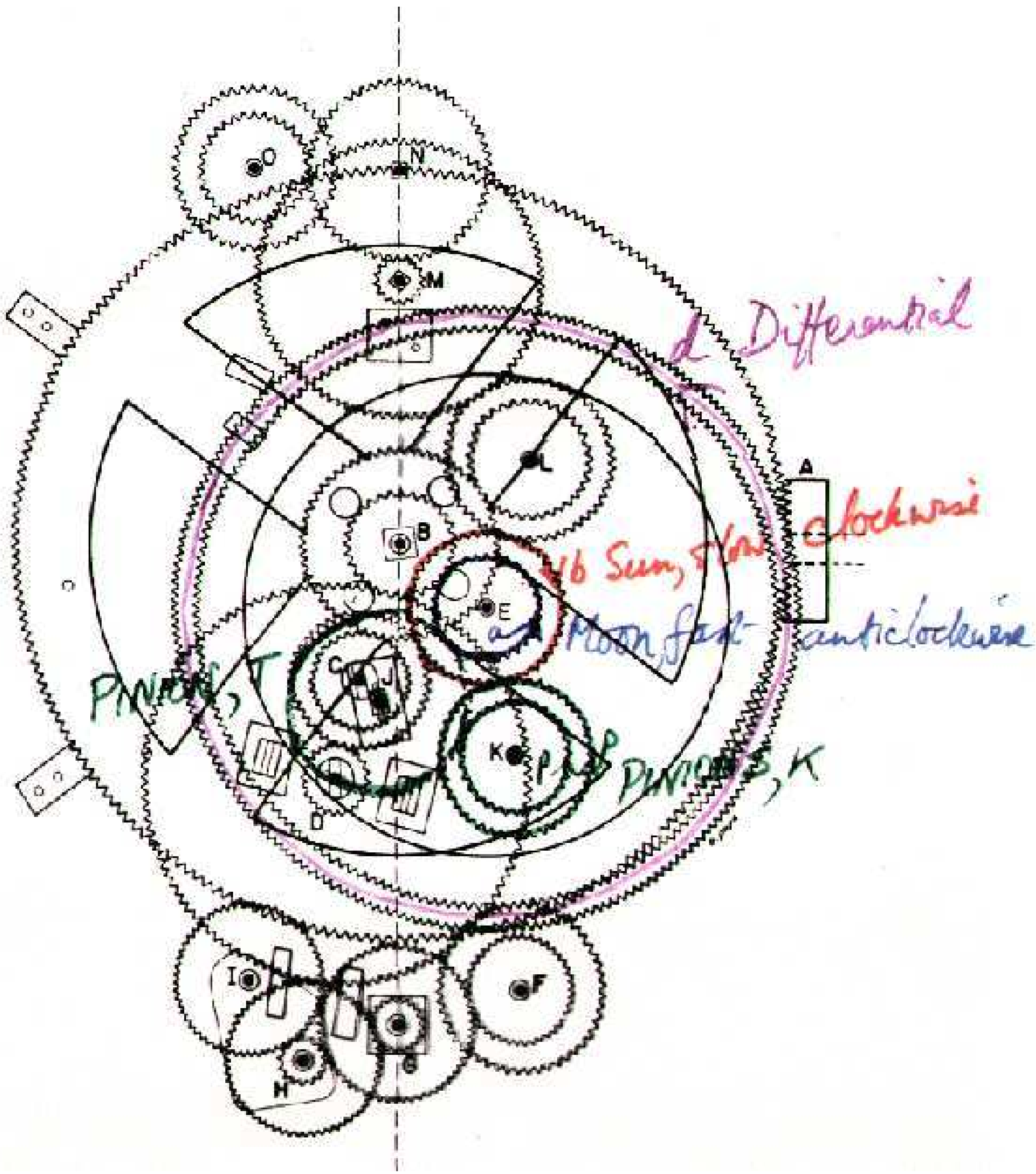
To exhibit the phases of the moon we need a gear going at speed $c = a - b$.

<u>Question</u>. Given gears going at speeds a, b how do you construct a gear going at speed $a - b$?

<u>Remark</u>. If all the gears have fixed axes it can't be done, because each gear determines the next.

Therefore one gear has to have a moving axis — call it a <u>pinion</u>. Call the gearwheel carrying that axis the <u>differential</u>.

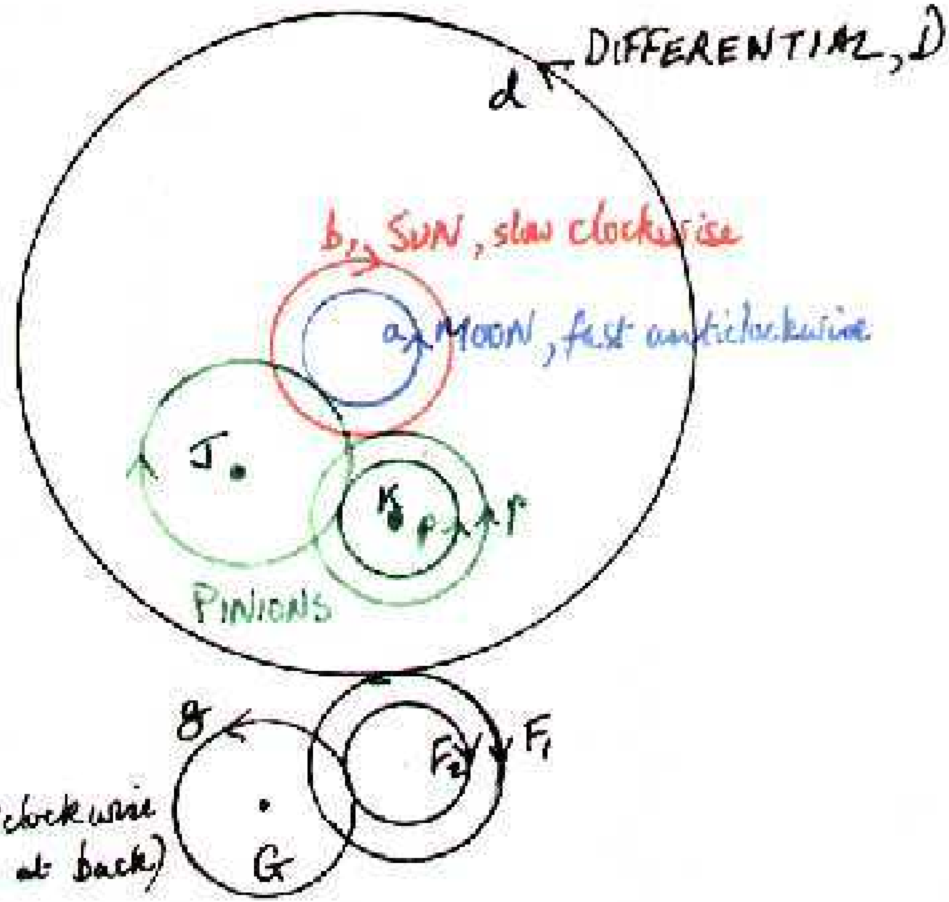

MOON POSITION
Moon clockwise
CRANK HANDLE
CONTRATE
SUN POSITION
Sun anticlockwise
DRIVE WHEEL
FRONT
BACK
4 YEAR DIAL
Moon anticlockwise
DIFFERENTIAL
Differential
Sun clockwise
Pinions K
Pinion J
SYNODIC MONTH
LUNAR YEAR



A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
Differential
Sun, slow clockwise
Moon fast anticlockwise
PINIONS, K

Small letters denote angular velocities.

Large letters denote number of teeth.



If $d = 0$ then $a = p = b$.

If $d \neq 0$ then $\begin{cases} a = p + d \\ b = p - d \end{cases}$

Subtract: $a - b = 2d$

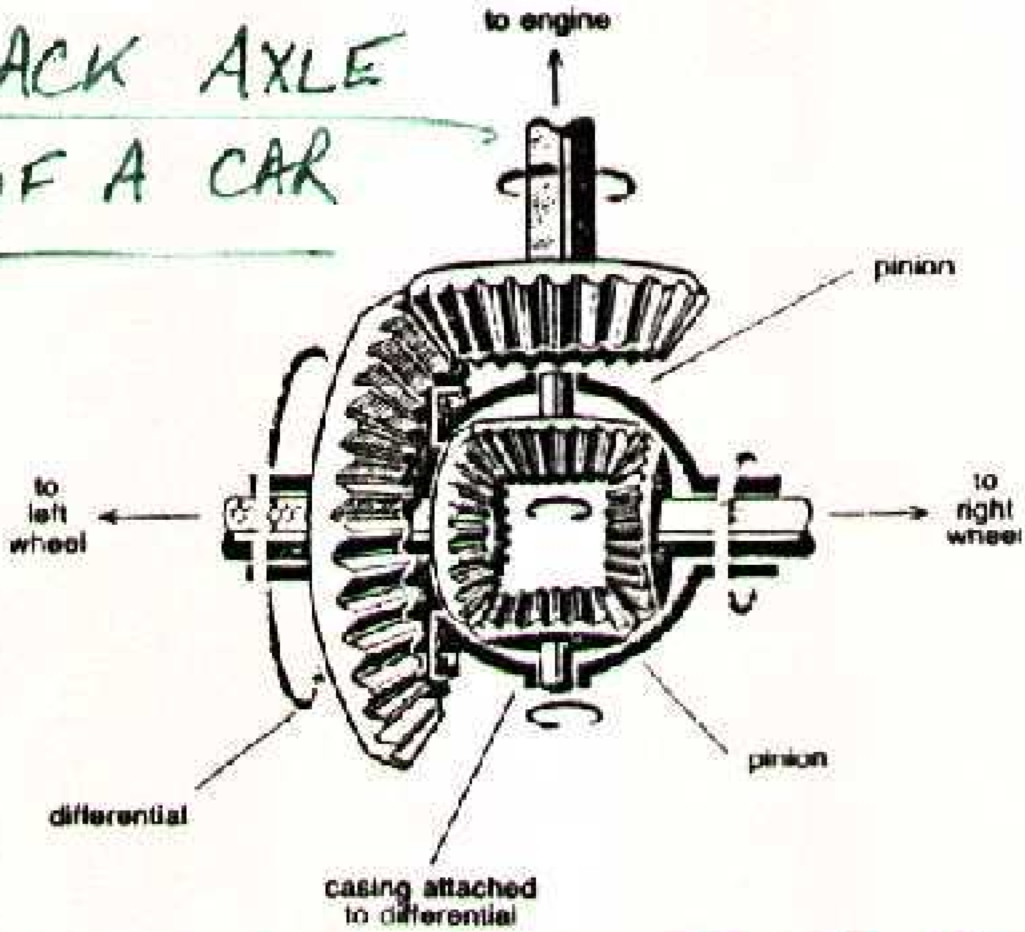
$\therefore \boxed{d = \frac{a-b}{2}}$ The differential goes at half the difference.

$$g = d \times \frac{D}{F_1} \times \frac{F_2}{G}$$

$$= \frac{a-b}{2} \times \frac{192}{48} \times \frac{30}{60}$$

$= a - b$. Hence the phases of the moon.

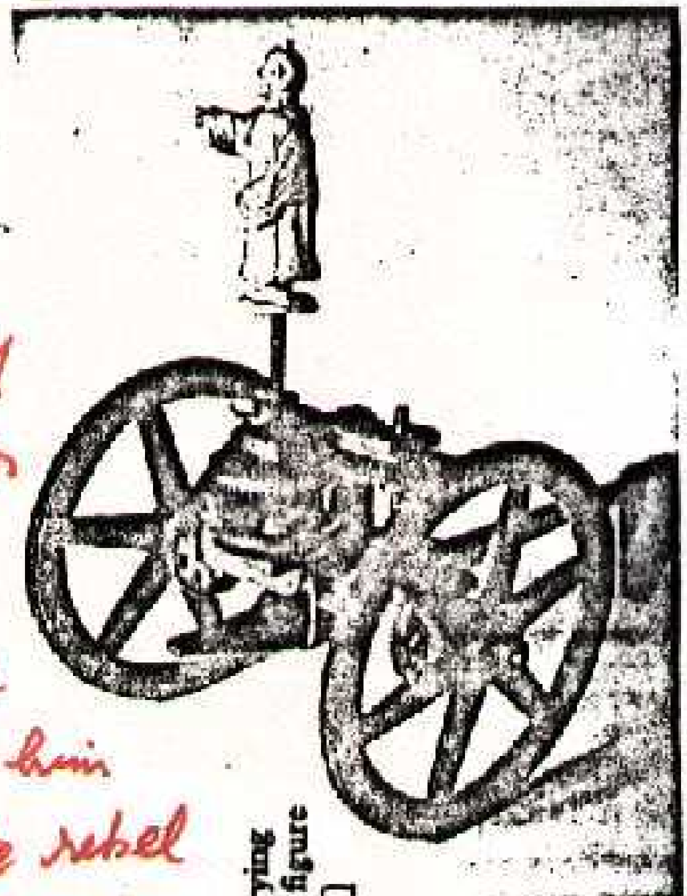
BACK AXLE OF A CAR

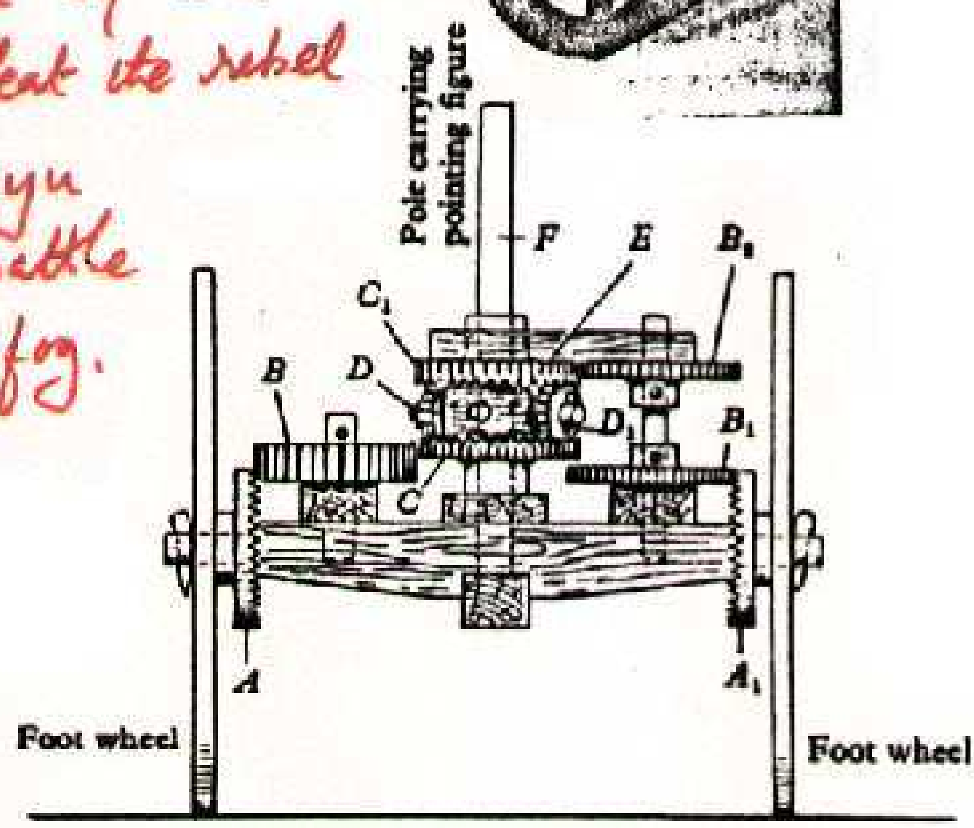


Invented by James Starley 1877

SOUTH POINTING CARRIAGE

Invented by the legendary Yellow Emperor Huang-ti in 2697 BC & used by him to defeat the rebel Ch'ih-yu in a battle in a fog.





HISTORY

① ASTRONOMERS (eg. Babylonians 1250 BC, Anaxagoras 430 BC)

Observed sun, moon, planets.

② MATHEMATICIANS (eg. Eudoxus 400 BC)

Invented notation to describe the motion for example epicycles.

③ ENGINEERS (eg. in Rhodes 80 BC)

Built mechanical models to simulate the motion, for example the Antikythera Mechanism

④ STUDENTS (eg. in Alexandria 50 AD)

Taught with mechanical aids.

⑤ PTOLEMY (in Alexandria 150 AD)

Proposed the Ptolemaic system of actual mechanical spheres rotating on spheres

⑥ PRIESTS (during the Dark Ages)

Insist that this is the mechanism, in order to support man being at the centre of the universe

⑦ LAYMEN (until Copernicus, Galileo & Newton)

Bamboozled into accepting the Ptolemaic system, with the sun going round the earth, for 1500 years!

HISTORY OF COMPUTING

Garbage in garbage out

1. Computers are invented by mathematicians, Who regard them with appropriate scepticism (GIGO)
2. Computers are developed by computer scientists, Who revere them.
3. Generations of students are reared on computers.
4. Neurologists begin to think that computer models are how the human brain works.
5. Social scientists begin to think that computer models are how the world works.
6. Politicians will order us to obey computers (in order to further their own ends).
7. The human race will be bamboozled into obeying computers for the next 2000 years.

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