Gears from the Greeks.

E.C. Zeeman.

1. Introduction.

The Antikythera Mechanism is the oldest and most sophisticated scientific instrument surviving from antiquity. It is a calendar computer made in Rhodes about 87BC. It contains 32 bronze gears, including a differential gear, and is accurate to 1 part in 40000. The Mechanism was discovered in a sunken ship by sponge fishermen in 1900, and seemed at first to be just a lump of calcified metal. It remained unexplained until 1972, when Derek de Solla Price [10] unravelled the mysteries of the interior by using X-rays. As a result, historians of science have been able to completely reassess the high technology of the ancient Greeks.

The ship was evidently travelling from Rhodes to Rome in about 70BC, and sank in a storm off the small island of Anthykithera (see Figure 1) - hence the name.

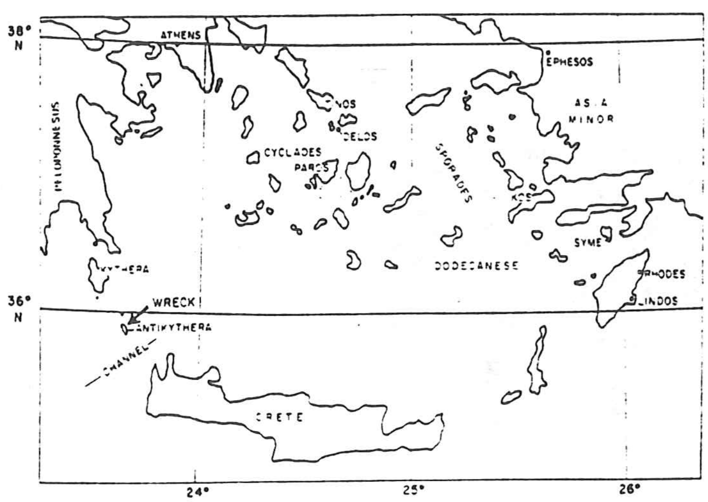

Figure 1: Map of the Aegean Islands [Price,10; Reproduced by permission of the American Philosophical Society].

There it lay until discovered by the sponge fishermen. They recovered a large hoard of bronze and marble statues, amphorae, pottery and coins, which was sent to the National Archaeological Museum in Athens, along with our uninteresting lump of calcified metal. After a while, however, the lump dried out and split into six fragments revealing traces of gear wheels (Figure 2) and graduated scales (Figure 3).

Figure 2: Fragment A showing gears [Price, 10; Photograph courtesy of National Archaelogical Museum, Athens].

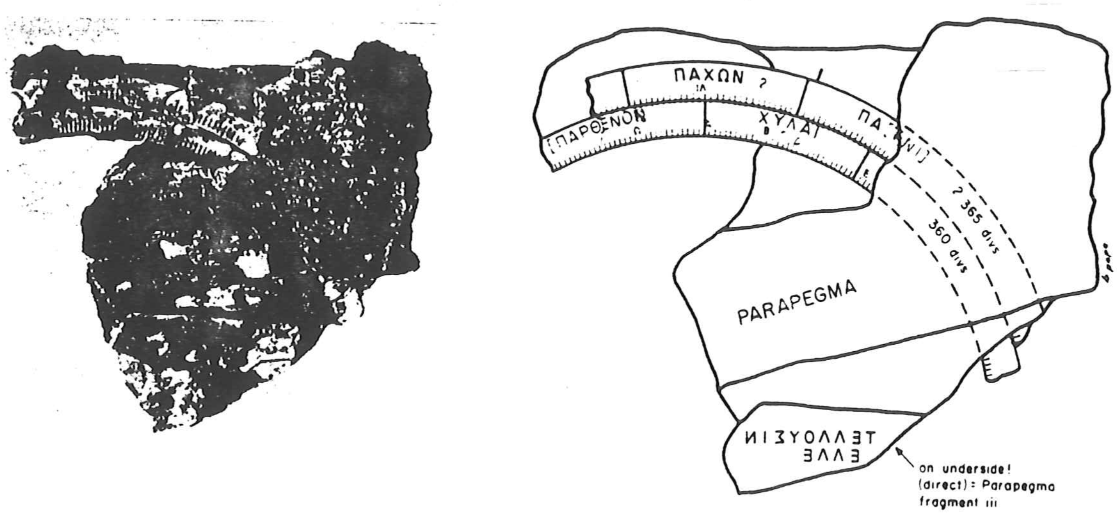

Figure 3: Fragment C1 showing graduated scales [Price, 10; Photograph courtesy of National Archaelogical Museum, Athens].

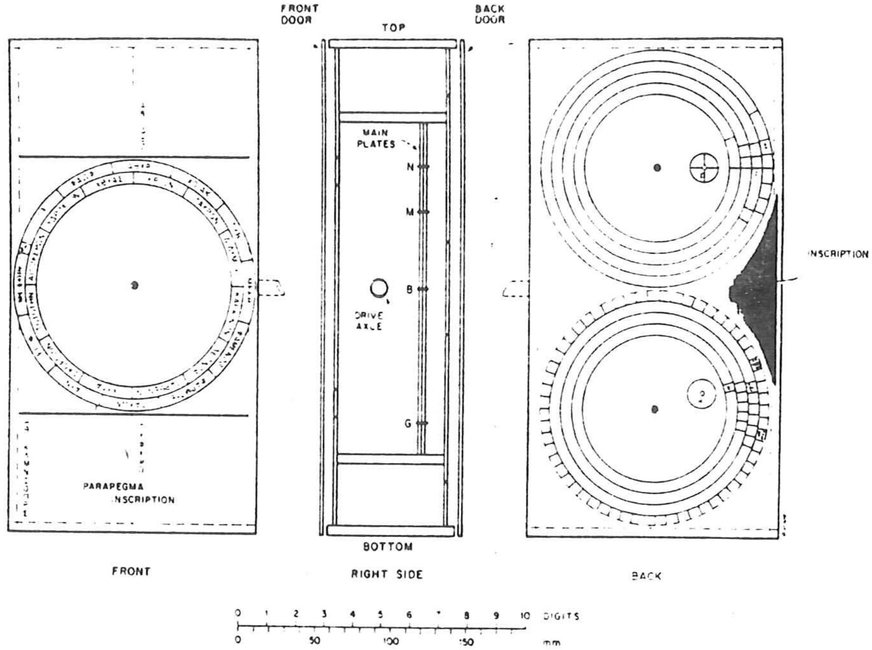

Figure 4: Outer casing and dials [Price, 10; Reproduced by permission of the American Philosophical Society].

Nobody understood what these meant until Price persuaded the physicist Char. Karakalos to take some X-ray photographs of the insides of the fragments (Figure 5). These revealed traces of the 32 gears, and after considerable detective work Price and Karakalos reconstructed the amazing gearing system shown in Figures 6 and 7.

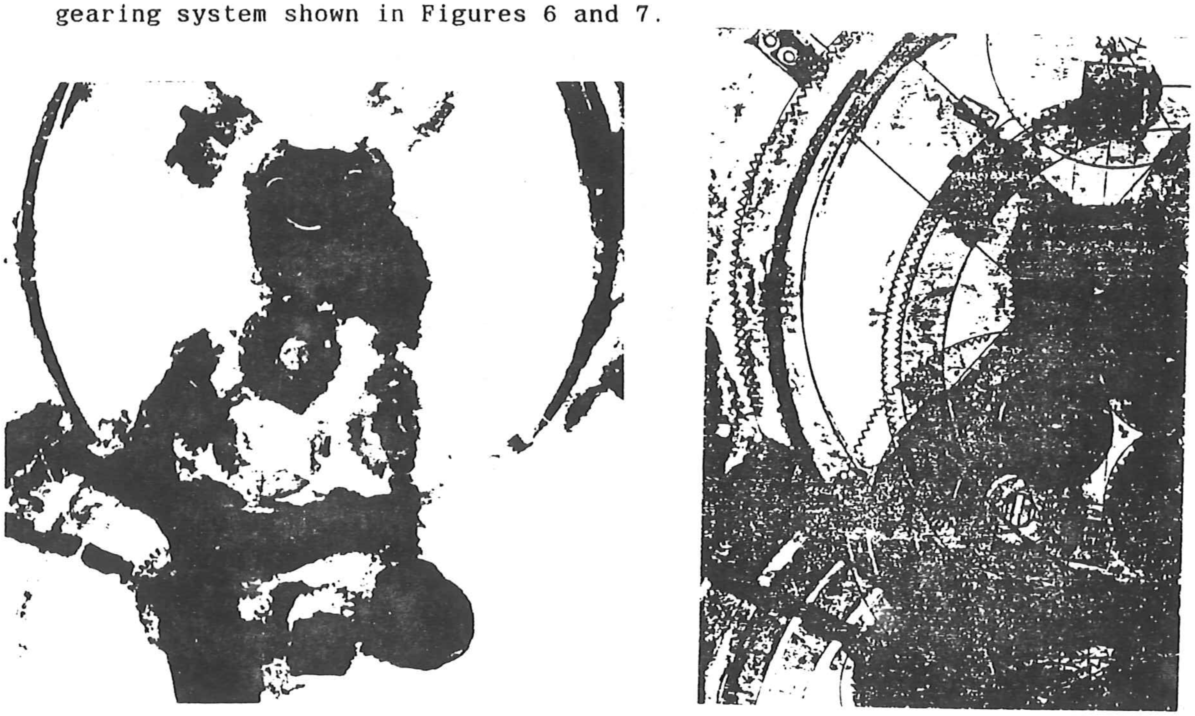

Figure 5: Radiographs of fragment A. In the enlargement on the right the gear teeth have been marked in ink for tracing the gear trains [Price, 10; Photograph courtesy of National Archaeological Museum, Athens].

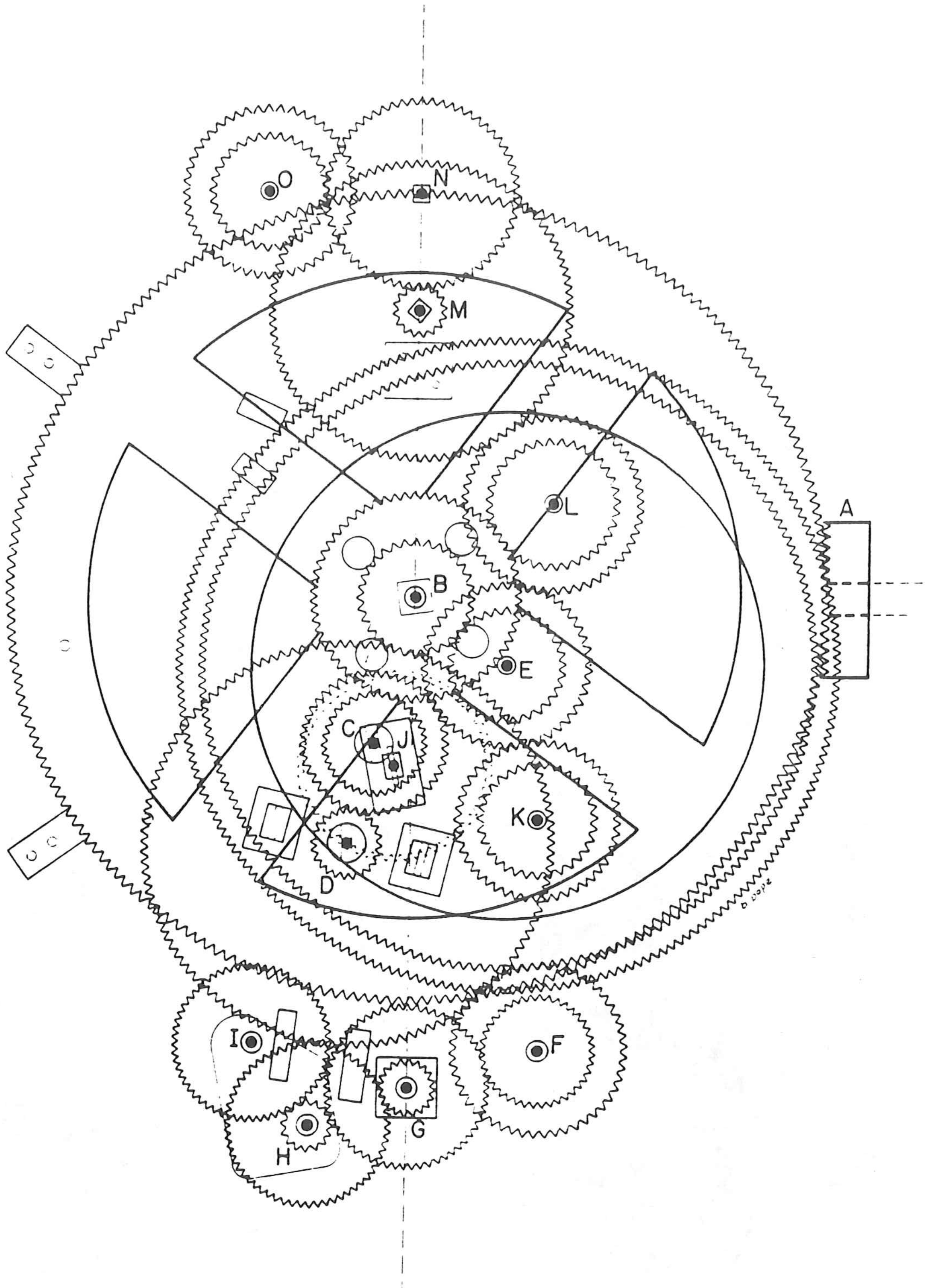

Figure 6: General plan of the complete gearing system [Price, 10; Reproduced by permission of the American Philosophical Society].

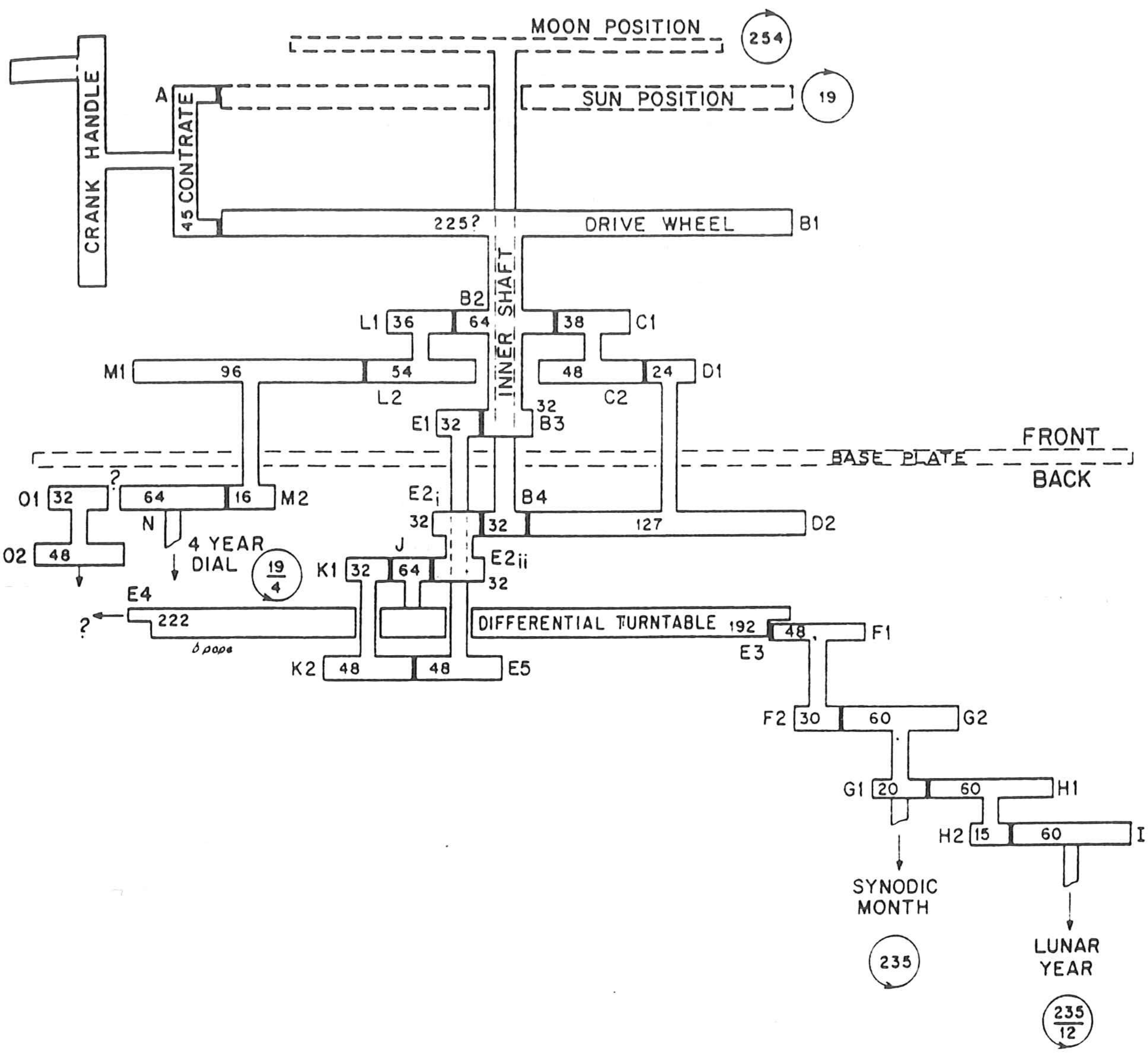

Figure 7: Sectional diagram of the complete gearing system [Price,10; Reproduced by permission of the American Philosophical Society].

Meanwhile they conjectured that the outside probably looked like the rectangular box shown in Figure 3, about 30cm high.

This life-size model that we have here tonight was kindly lent to us by the Smithsonian Museum in Washington. Some of you may have already seen it in the fascinating little exhibition on Early Gearing [3] mounted by Judith Field and Michael Wright in the Science Museum this year. It is an astonishing fact that of all the examples of precision gearing made before AD1200 only two are known to have survived, probably because all the others eventually got broken and were melted down again as valuable scrap metal. One is the Anthikythera Mechanism and the other is a brass Byzantine sundial-calendar made about 600 years later [4], which was acquired by the Science Museum in 1983. We have here tonight a beautiful working model of the latter reconstructed by Michael Wright. Although it is impressive, it is not to be compared with the complexity and ingenuity of the Antikythera Mechanism.

I am indebted to Richard Gregory [8] for first drawing my attention to the Antikythera Mechanism. I have had a great deal of pleasure in finding out how it worked, and would like to share some of its secrets with you tonight. We must begin by asking how do we know that it was a calendar computer?

2. The front face.

The first step in the detective story is given by the fragment in Figure 3, from which we can reconstruct the front dial, shown on the left in Figure 4. The inner scale is divided by 360^0 into the twelve signs of the Zodiac. Here ΠΑΡΘΝΟΝ corresponds to Virgo (running from 23 August to 22 September), and ΧΥΛΑΙ corresponds to Libra (running from 23 September to 22 October). We conjecture that the motions of the sun and moon against the background of fixed stars were represented by hands (or other indicators) moving clockwise round the face, the sun hand going round once a year, and the moon hand once a month. In other words we have two hands on a single dial, one going about 12 times as fast as the other. It may be no accident that this bears a resemblance to the modern clockface, even though the latter is measuring hours and minutes rather than years and months, because there may have been a continuous tradition of precision gearing from that day to this.

But why did the designer choose the hands to go clockwise rather than anticlockwise? The answer is that against the background of fixed stars in the Northern sky the sun and moon go clockwise round the circle of the Zodiac, and so the front face of the Antikythera Mechanism is merely a symbolic picture of the sky. And maybe this is why our clocks today go clockwise.

Meanwhile the outer scale is divided into 365 days, comprising the 12 equal months (each of 30 days) of the Egyptian-Greek calendar together with the 5 special epagomenal days necessary to complete the year. Of course it actually takes the sun 365¼ days to go round the Zodiac, and so every leap year the outer scale has to be moved back one notch to compensate. That is why the outer scale slides on the inner scale. The Egyptian-Greek calendar had already been running for several centuries so people were very familiar with the convention of moving the calendar back a notch every leap year. In fact Price suggests that the upper dial on the back face may have indicated leap years for this purpose.

3. The sun-moon gear train.

What is the evidence for the sun and moon hands? For this we have to look at the gear train

$$B2 \rightarrow C1 \Rightarrow C2 \rightarrow D1 \Rightarrow D2 \rightarrow B4 \;,$$

shown in Figures 6,7, and 8, where a single arrow denotes "drives" and a double arrow denotes "is fixed to". We can calculate the gear ratio of this gear train from the numbers of teeth in the gearwheels shown in Figure 7, which Price and Karakalos [10] obtained from the radiographs (as in Figure 5):

$$\text{gear ratio} = \frac{64}{38} \times \frac{48}{24} \times \frac{127}{32} = \frac{254}{19} = 13.36842\ \ldots$$

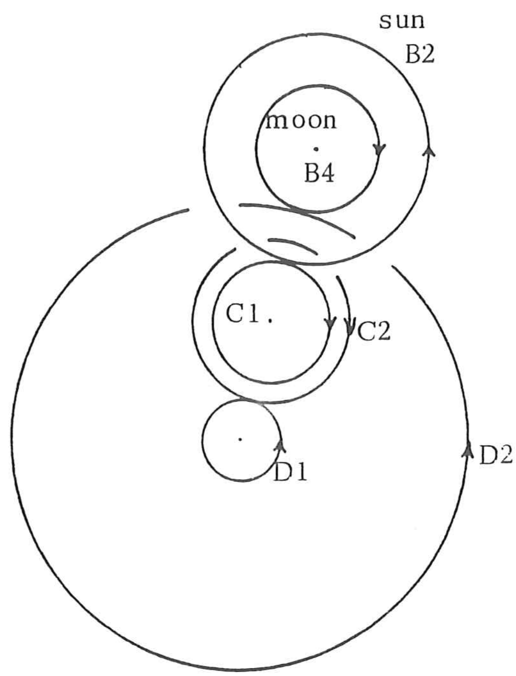

Figure 8: The sun-moon gear train.

Now the mean periods taken by the sun and moon to go round the Zodiac are called the sidereal year and month, and by modern measurements

$$\frac{\text{sidereal year}}{\text{sidereal month}} = \frac{365.25636\ldots\text{days}}{27.32167\ldots\text{days}} = 13.36874\ldots$$

Therefore if this gear train does represent the relationship between the sun and moon then it is accurate to 1 part in 40000, which is very strong evidence in its favour.

Notice in Figure 8 that if B4 represents the moon going clockwise, then B2 will represent the sun going paradoxically *anticlockwise*, in the wrong direction. We shall discover the reason for this when we come to analyse the differential gear in Remark 2 of Section 6 below. Meanwhile in Figure 7 it can be seen that the designer has compensated for it by adding a contrate gear A meshing with B1, that reverses the direction, and drives a sun wheel going clockwise in the right direction.

4. A digression on accuracy.

How did the Greeks manage to find so accurate an approximation to the ratio of year to month? Not only is $\frac{254}{19}$ very accurate, but it is in fact the ***best possible*** approximation by any rational number* with denominator less than 80. This level of accuracy corresponds to measuring the mean length of the month to the nearest minute. But the Greeks had no instruments that could measure time so precisely. Nor would it have been possible to observe the position or the phases of moon with such precision, to say nothing of having to take the average of several readings because of the variations in the length of the month. And even if they had been able to measure accurately they did not have real numbers or decimals with which to express the results, nor any technique of division of real numbers with which to calculate the ratio. So how on earth did they do it?

Here we are not talking about the designer of the Antikythera Mechanism in the 1^{st} century BC, but about the Greek and Persian astronomers in the 5^{th} century BC, or possible the Babylonian astronomers much earlier. For example Meton introduced a 19-year cycle, called the ***Metonic cycle*** [7,9], in Athens in 431 BC, which is still used today for determining the date of Easter. This cycle is based on the ***Metonic ratio***:

$$19 \text{ sidereal years} = 235 \text{ synodic months},$$

where the mean synodic month is the average period between two new moons (or equivalently between two full moons). The synodic month is roughly 29.5 days, as opposed to the sidereal month which is roughly 27.3 days. We need to prove a couple of lemmas to verify that the gear ratio used in the Antikythera Mechanism is in fact equivalent to the Metonic ratio.

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* A rational number is the ratio of two integers $\frac{p}{q}$.

Let S = sun, E = earth, M = moon. Let

a = angular velocity of M about E

b = angular velocity of S about E (equivalently E about S)

c = relative angular velocity of M relative to the direction ES.

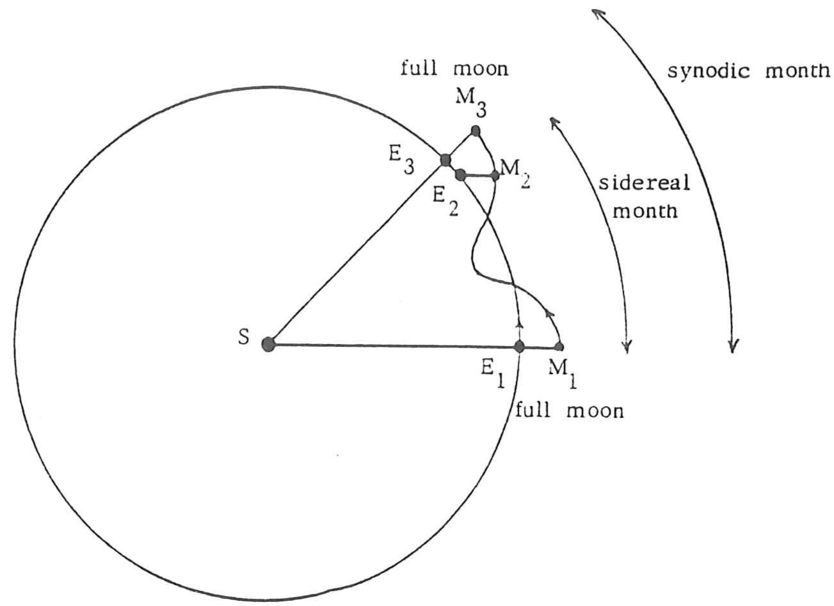


Figure 9: The sidereal month is the period between two positions where EM points in the same direction, and the synodic month is the period between two full moons.

<u>*Lemma 1.*</u> $a - b = c$.

<u>*Proof:*</u> by definition of relative velocity.

<u>*Lemma 2.*</u> $$\frac{\text{sidereal year}}{\text{sidereal month}} = 1 + \frac{\text{sidereal year}}{\text{synodic month}} .$$

<u>*Proof:*</u> The sidereal month $= \frac{1}{a}$

sidereal year $= \frac{1}{b}$

synodic month $= \frac{1}{c}$.

$$\therefore \quad \frac{\text{sidereal year}}{\text{sidereal month}} = \frac{a}{b}$$

$$= \frac{b+c}{b} \quad \text{by Lemma 1}$$

$$= 1 + \frac{c}{b}$$

$$= 1 + \frac{\text{sidereal year}}{\text{synodic month}} \, .$$

Substituting the Metonic ratio

$$\frac{\text{sidereal year}}{\text{sidereal month}} \triangleq \frac{235}{19} \, ,$$

we deduce

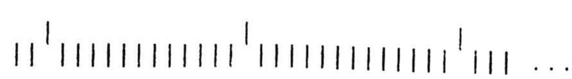
$$\frac{\text{sidereal year}}{\text{sidereal month}} \triangleq 1 + \frac{235}{19} = \frac{254}{19} \, ,$$

which is the sun-moon gear ratio of the Antikythera Mechanism.

So the designer was merely using a well-known ratio. But the question still remains: how did Meton or his Babylonian predecessors manage to find so accurate an approximation? I would like to give an algorithm to show how it could have been done, and then prove a theorem to show why this algorithm gives the correct answer.

5. <u>The algorithm.</u>

Imagine you are an early astronomer, wanting to know how many months there are in a year. The simplest approach is just to count the number of new moons per year. Of course you have to know when the year begins, and one way to do this is to choose a particular bright star, and define "new year" to be the first day of the year on which that star is visible at sunset. You then record the sequence of new moons and new years, by cutting notches on a stick, for example, or making marks on a wax tablet.



You notice that there are sometimes 12 and sometimes 13 new moons in a year, so you write down the sequence of numbers of new moons per year for many years:

12,13,12,12,13,12,13,12,12,13,12,12,13,12,12,13,12,13,...

After some time you begin to notice a new pattern because there are always 2 or 3 steps from each 13 to the next. So your write down the sequence of numbers of steps from each 13 to the next:

3,2,3,3,3,2,3,3,2,3,3,3,2,3,3.2,3,3,2,3,3.3,2,3,...

You then notice there are always 1 or 2 steps from each 3 to the next, so you write down that sequence

2,1,1,2,1,2,1,1,2,1,2,1,2,1,1,2,...

Same thing again:

3,2,3,2,2,3,...

And again:

2,3,...

This is as far as you can get if you (and your predecessors) had only collected 65 years of data, because each sequence is shorter than the previous one. Now pick out the smaller number in each sequence:

12,2,1,2,2

and assemble them into a continued fraction

$$12 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2}}}}$$

The work it out and lo behold you arrive at the magic Metonic ratio $\frac{235}{19}$.

Note that there are other more ad hoc ways of obtaining the Metonic ratio [e.g. 7] but this algorithm is a *systematic* way of obtaining best possible approximations, within bounds that can also be calculated. Moreover the procedure is typical of Greek mathematics, since it is closely related to the Euclidean algorithm, or anthyphairesis [5,6]. In the Appendix we prove a theorem to explain why the algorithm works.

6. The differential gear.

Let us now return to the Antikythera Mechanism and look at the back plate (on the right of Figure 4). Price [10] suggests that the lower dial registers the phases of the moon. Indeed one would expect the latter to be an obligatory feature of any sophisticated calendar computer. We already have gears in the sun-moon gear train rotating with the speeds a,b of the moon and sun (see Section 3 above). What is needed to register the phases of the moon is another gear rotating with the relative speed a-b. Therefore the mechanical problem facing the designer was: *given two gears rotating with speeds a,b how to construct a third gear rotating with speed a-b?*

If all the axes of all the gear wheels are fixed then the solution is impossible, for the motion of any gear in a gear train is completely determined by that of its predecessor. In other words no gear can be simultaneously driven by two other gears. Therefore to solve the problem it is necessary to have some gears with moving axes. In other words *it is necessary to invent the differential gear*. And this is the remarkable innovation contained within the Antikythera Mechanism, made in 87BC.

Let us introduce the following terms. Any gear whose axis moves is called a *pinion*. The gear carrying the axes of the pinions is called the *differential*.

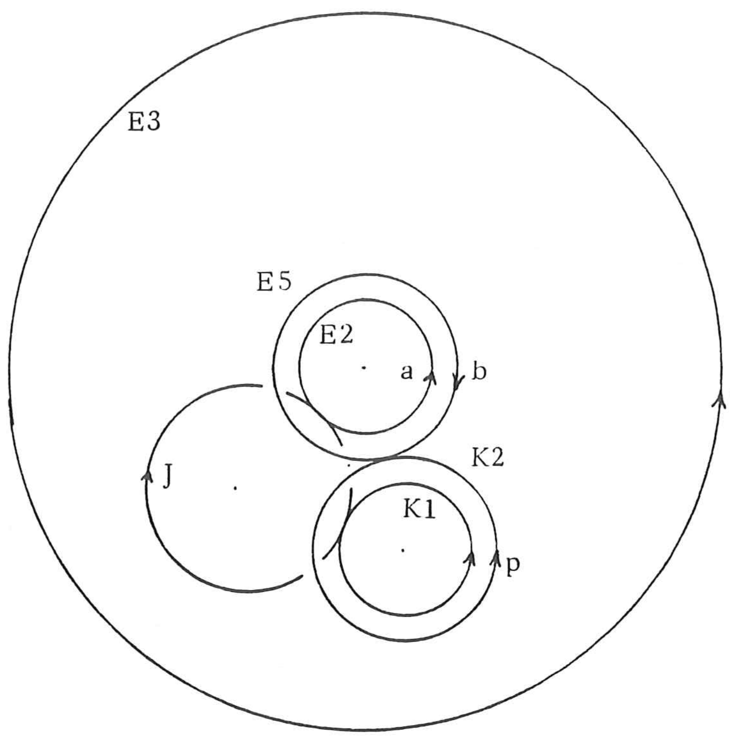


Figure 10: The differential gear E3 carries the axes of the pinions J,K.

In Figures 6,7 and 10 the differential is E3 and the two pinions are J and K. In fact K consists of two gears K1 and K2 fixed together. In Figure 7 it can be seen that the gears on the E-axis mesh with those on the B-axis, whose speeds are already determined by the sun-moon gear train (see Section 3 above). Therefore E2 rotates anticlockwise with the moon speed, a, while E5 rotates clockwise with the sun speed, b.

Let

d = speed anticlockwise of the differential E3

p = speed anticlockwise of the pinion K.

Lemma 3. $a = p+d$

$b = p-d.$

Proof. Fix $d = 0$.
Then $a = p$ because E2 and K1 are the same size and both mesh with J.
Also $b = p$ because E5 and K2 are the same size and mesh together.
If the differential E3 is now rotated with speed d then this is added to a, and subtracted from b, proving the lemma.

Corollary. $d = \frac{a-b}{2}$.

Proof. Subtract the equations in Lemma 3.

In other words the differential is being driven anticlockwise at half the speed that we want. Therefore if we were to mesh the differential with a gear half its size, the latter would rotate clockwise at the desired speed a-b. However there is one snag: we have been looking at everything from the *front*, and we want to register the phases of the moon on the ***back*** dial, and therefore we need a gear that rotates clockwise when seen from the back, in other words anticlockwise when seen from the front. The designer solved the problem by taking a two-step gear train off the differential (which incidentally had the additional advantage of employing smaller gears that could be more neatly packed into the box):

$$E3 \rightarrow F1 \Rightarrow F2 \rightarrow G2$$

with gear ratio

$$\frac{192}{48} \times \frac{30}{60} = 2 \text{ , as desired.}$$

Hence G provides the necessary gear for registering the phases of the moon on the lower dial of the back plate.

Remark 1. The reason that the designer had to put the differential on a

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separate axis E, rather than on the main axis B, is topological. If he had tried to take the sun and moon drives directly off B3 and B4 (rather than indirectly via E1 and E2) then the pinions would have snarled the sun-moon gear train.

Remark 2. We can now see why the designer made the sun and moon gears in the original sun-moon gear train go paradoxically in opposite directions; it was so that the differential could register the difference $\frac{a-b}{2}$ rather than the average $\frac{a+b}{2}$.

Remark 3. The differential in the back axle of a car contains exactly the same principle (see Figure 11). Here the differential is driven by the engine and runs at the average speed of the two back wheels, allowing them to corner at different speeds.

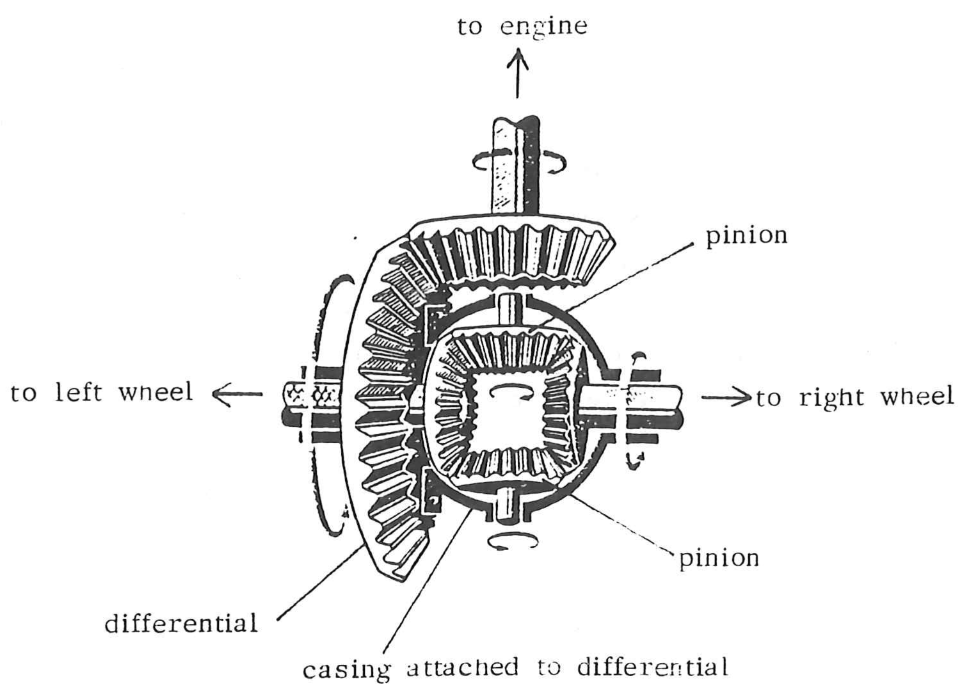

Figure 11: The differential in the back axle of a car.

The inventor of the modern differential gear was James Starley in 1877, according to the engraving on his statue in Coventry. In fact, he is one of a number of people who have rediscovered the principle since the original Greek invention. But the way he discovered it is worth telling [13]. He was an inventor of bicycles, amongst other things, and one day he invented a quadricycle by bolting two bicycles together. He and his son decided to test the new machine by cycling from Coventry to Birmingham for tea. Their progress caused some amusement, and when they got to Blacklow Hill Starley was reluctant to get off and push the machine up the hill, so he said to his son "Wire it up, lad", meaning put on some speed. So his son, who was a strapping lad, duly accelerated his side of the machine causing the whole apparatus to swerve into the ditch and pitchfork them both into a bed of nettles. Whereupon Starley clambered out and sat on the edge of the ditch lost in thought, until suddenly he cried "Eureka" for he had discovered the differential gear. Except that he arranged to have it the other way round: instead of one engine driving two wheels at different speeds, he had two engines driving one back axle at different speeds. Thus while his son was pedalling vigorously he himself was able to maintain a more leisurely pace without fear of being thrown into the ditch again.

7. <u>The crank handle.</u>

Price did not claim to have solved all the gearing of Antikythera Mechanism. Indeed the main weakness of his reconstruction is the suggestion that the whole machine be driven by a crank handle attached to the contrate gear A (see Figure 7), for this involves a substantial step-up mechanism. It is like trying to alter a clock by pushing the hour hand, and expecting this to drive the minute hand twelve times as fast. I mentioned this weakness in a lecture at Chelsea a couple of years ago and it was picked up by Michael Wright, himself an experienced gear-maker, and he confirms that the Smithsonian model does not work for this reason. He passed the problem on to Allan Bromley, who has subsequently developed some interesting modifications [1] of Price's reconstruction, by driving the whole machine with a more realistic step-down mechanism from the other end, via the differential. Eventually the crucial test will be to make a working model.

8. Conclusion.

I would like to conclude by telling a cautionary tale. Let us try and place the Antikythera Mechanism within the global context of ancient Greek thought. Firstly came the astronomers observing the motions of the heavenly bodies and collecting data. Secondly came the mathematicians inventing mathematical notation to describe the motions and fit the data. Thirdly came the technicians making mechanical models to simulate those mathematical constructions, like the Antikythera Mechanism. Fourthly came generations of students who learned their astronomy from these machines. Fifthly came scientists whose imagination had been so blinkered by generations of such learning that they actually believed that this was how the heavens worked. Sixthly came the authorities who insisted upon the received dogma. And so the human race was fooled into accepting the Ptolemaic system for a thousand years.

Today we are in danger of making the same mistake over computers. Our present generation is able to view them with an appropriate skepticism when necessary. But our children's children may be brought up within a society so dominated by computers, that they may actually believe this is how our brains work. We do not want the human race to be fooled again for another thousand years.

9. Appendix.

Suppose that we are given two periodic events, A and B (such as new years and new moons). Suppose period A > period B, and let $x = \frac{\text{period A}}{\text{period B}}$.

Therefore $x > 1$. Assume that there are no coincidences*, in other words no A coincides with any B.

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* This is a reasonable assumption because if the sequence starts with an A then the whole sequence is determined by the time interval before the next B, and since there are uncountably many possible time intervals, of which only countably many can lead to a concidence, almost all sequences will have no coincidence.

Let S be the sequence of numbers of B's between successive A's. We call S an *unfolding* of x. Write

$$x = n + r ,$$

where n is the integer part of x, and r the remainder, $0 \leq r < 1$.

<u>*Lemma 4*</u> If x is an integer (in other words $r = 0$) then S is a sequence of all n's.

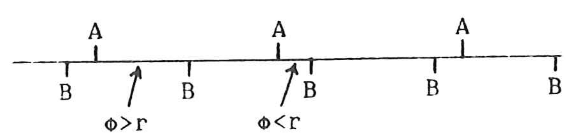
<u>*Proof.*</u> Since period A is n times period B, there are exactly n B's between any two consecutive A's.

<u>*Lemma 5.*</u> If x is not an integer (in other words $r > 0$) then S is a sequence of n's and n+1's.

<u>*Proof.*</u> Given any A let

$$\Phi = \frac{\text{time interval before the next B}}{\text{period B}} .$$

We call Φ the *phase-lag*.
Notice $\Phi \neq r$, otherwise the next A would be a coincidence. Then there are exactly n or n+1 B's before the next A according as to whether $\Phi > r$ or $\Phi < r$.



<u>*Definition.*</u> If x is not an integer (in other words r>0) we make the following definitions.
The *derived number* x_1 of x is defined by $x_1 = \frac{1}{r}$.

The *derived sequence* S_1 of S is defined to be the sequence of numbers of steps in S from each n+1 to the next.

Lemma 6. S_1 is an unfolding of x_1.

We postpone the proof of Lemma 6 for the moment.

If x_1 is not an integer, then by Lemmas 5 and 6 we can define the derived x_2, S_2 of x_1, S_1, and so on. We obtain a sequence of derived numbers

$$x, x_1, x_2, x_3, \ldots$$

and a sequence of derived sequences

$$S, S_1, S_2, S_3 \ldots$$

such that S_i is an unfolding of x_i, for each i. There are two possibilities depending upon whether x is rational or irrational.

(i) If x is rational then after a finite number of steps some x_k is an integer, and so we have to stop.

(ii) If x is irrational then no x_k is an integer, and so we never have to stop.

Let n_i denote the integer part of x_i, for each i.

Theorem. The continued fraction expansion of x is given by

$$x = n + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \ldots}}$$

If x is rational the continued fraction stops at n_k, and if x is irrational it never stops.

Remark. The advantage of the Theorem is that if we do not know x, but do know S, then we can compute the derived sequences $S, S_1, S_2, \ldots$ and use Lemma 6 to obtain $n, n_1, n_2, \ldots$, and then use the Theorem to calculate x to any desired accuracy by truncating the continued fraction appropriately.

Proof of the Theorem.

By definition of derived number

$$x = n + \frac{1}{x_1}$$

$$x_1 = n_1 + \frac{1}{x_2}$$

$$x_2 = n_2 + \frac{1}{x_3}$$

$$\vdots$$

Therefore

$$x = n + \cfrac{1}{n_1 + \cfrac{1}{x_1}} = n + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{x_2}}} = \text{desired expansion.}$$

If x is rational then $x_k = n_k$, and so the continued fraction stops at n_k.

Proof of Lemma 6.

Represent the A-events by vertical lines in the plane, spaced a unit distance apart, and the B-events by horizontal lines, also spaced a unit distance apart as illustrated in Figure 12. Taking period A as the unit of time, we have to proceed with a constant velocity (1,x) in order to cross the A-lines with period A and the B-lines with period B. In other words we must proceed with constant speed along a straight line X with slope x. The position of X is determined by the initial phase-lag between the first A and the next B. Meanwhile X determines an unfolding S of x as follows. As X crosses the vertical and horizontal lines we can read off the sequence of events; for example in Figure 12 the line X determines the sequence of events

ABBABABABBAB... .

Hence the unfolding S of x is

2,1,1,2,... .

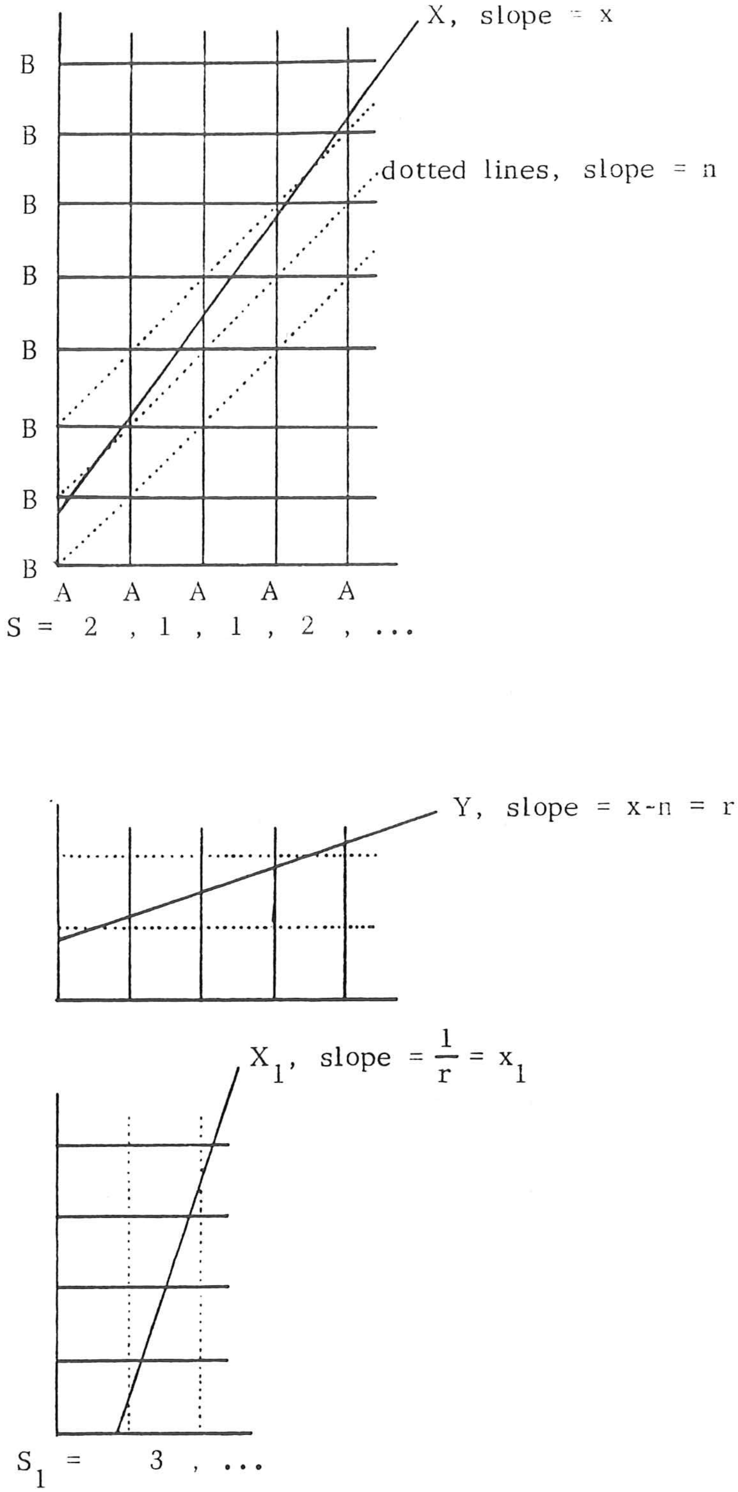


Figure 12: Example illustrating the proof of Lemma 6.

Here $x = {}^4/_3$, approximately.

Draw in dotted lines of slope n through the intersections of the A-lines with the B-lines and observe where they cross X. In fact there is exactly one crossing in each A-interval containing n+1 B's, and none in the other A-intervals, because the condition for X to cross a dotted line after an A with phase-lag ϕ is $\phi<r$, which is exactly the same as the condition for n+1 B's.

Now apply a sheer map to the plane that leaves the A-lines vertical and makes the dotted lines horizontal (see Figure 12). This has the effect of reducing the slope of all lines by n. In particular the image Y of X has slope $x-n = r$. Now reflect the plane in the diagonal so that the A-lines become horizontal, and the dotted lines vertical. Then the image X_1 of Y has slope $\frac{1}{r} = x_1$.

Consider the derived sequence S_1 of S.

Each term of S_1 = number of steps of S between two n+1's
= number of A-lines between two X-crossings of dotted lines
= number of A-lines between two Y-crossings of dotted lines
= number of horizontal lines between two X_1-crossings of dotted lines
= a term in the unfolding of x_1 determined by X_1 .

Hence S_1 = the unfolding of x_1 determined by X_1.

This completes the proof of Lemma 6 and the Theorem.

Remark 1. The Theorem is subtle because it gives ***metric information from order information***, which is unusual. Moreover it is valuable for applications because order information is easy to observe (like recording the sequence of new years and new moons) whereas metric information is difficult to observe (like the difficulty of measuring the mean year and month correct to 1 part in 40000). Thus the algorithm enables us to bypass the difficulty.

Remark 2. The Theorem is interesting because it captures an ***invariant***. For there are uncountably many different unfoldings depending upon the uncountably many possible initial phase-lags. But they all have one property in common, namely x, and this is precisely what the theorem captures.

Remark 3. The converse of the theorem, going from x to the sequence which starts from a coincidence, was discovered over a century ago by Christoffel [2] and Smith [12]. However by Remark 1 this is less useful for applications.

Remark 4. The algorithm is self-correcting. For suppose a new moon was so close to a new year that the observer made an error by recording them in the wrong order. This would show up in some derived sequence by producing an alien term not equal to either n or n+1, which would then swiftly lead to the detection and correction of the error.

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I am indebted to David Fowler for first suggesting the problem to me in connection with his reconstruction of pre-Euclidean ratio theory and theoretical astronomy [5,6]. Whenever he suggests that the Greeks could not have proved some conjecture or other I confess it is like a red rag to a bull, and I am tempted to try and show how they might possibly have done it. My initial proof [14] of Lemma 6 was much more in the spirit of Greek mathematics, reconstructing the derived sequence by appropriately chopping up the line in the manner of the Euclidean algorithm. However this method of proof is less appealing to the modern mathematical mind, and when I explained it to Caroline Series she produced the much lovelier geometrical proof above. Furthermore she used it to develop new results in hyperbolic geometry and number theory [11].

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