FLUID MECHANICAL DESIGN OF THE POTENTIAL FLOW TURBINE

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Abstract

This paper presents the software package developed at Fluid and Power Research Institute (FPRI) for the fluid mechanical design of the wicket gates and the runner blades of the Potential Flow turbine [1]. This software package is based on the correct formulation of the design problem. The design problem can not be correctly formulated for Francis and Kaplan turbines with conventional wicket gates, since in these turbines the flow coming to the runner is neither axisymmetric nor potential. The package is based on a quasi-three-dimensional approach. All programs in the package are highly accurate from the computational point of view. The application of FPRI fluid mechanical software directly leads to the best solution from the point of view of efficiency and cavitation. The experimental data support the predictions made by the FPRI software [2,3].

Introduction

There are three very important considerations necessary for the success of the fluid mechanical design of the turbine:

(i) Correct formulation of the design problem, the problem of the fluid mechanical design of the runner blades for given flow at the entrance.

(ii) Use of the approach to the design problem which directly leads to the optimal solution from the point of view of efficiency and cavitation for the given initial design parameters.

(iii) High accuracy of numerical computations included in the software package used for the solution of the problem.

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Let us analyze, from the point of view of these considerations, the software being used by major turbine manufactures for the the fluid mechanical design of Francis and Kaplan turbines :

(i) In Francis and Kaplan turbines with cylindrical radial wicket gates, variation of the whirl, $(V_u R)_i$, along the span of the wicket gate generates a vortex wake leaving the trailing edge of each gate along streamlines of absolute flow [4]. Due to these vortex wakes the flow at the entrance to the runner is not axisymmetric and, therefore, the design problem is ambiguous. Indeed, the relative flow at the entrance to the runner in this case is unsteady and presents a problem. It is not clear for which position of the runner with respect to the wicket gates the runner blade has to be designed.

(ii) There are two completely different approaches to the design problem. The first is based on the method of singularities directly leading to the optimal design from the point of view of efficiency and cavitation for given values of turbine head, H_t, flow rate, Q, rotational speed, N, and velocity distribution at the entrance to the runner [2]. The second approach is based on the determination of the flow in the runner with specified geometry of the blades for given values of H and N. In order to achieve the required value of Q the variation of geometry is used. It is clear that, since the initial geometry of the blades was not accepted from fluid mechanical considerations, the second approach does not lead to the optimal solution. The major turbine manufacturers use the second approach for the design of both Kaplan and Francis runners (the first approach has been used by Voith for Kaplan design), and this geometrical approach limits their capabilities to design the most efficient equipment.

(iii) The mathematical accuracy of the solution based on finite elements sometimes could be very low. For example, the application of well known finite element program ANTONY to such a relatively simple problem of determination of the axisymmetric flow in the turbine leads to high errors in velocity computation. Even larger errors in velocity and pressure were obtained using a finite element technique program (developed by Dr. C. Taylor et al.) for the solution of Navier-Stokes equations in application to a problem of the fully developed viscous flow in a pipe at high Reynolds numbers [5]. Generally speaking, a majority of the papers written on application of the numerical methods to the flow analysis in the turbine directly compare experiment with the numerical results without any assessment of accuracy of the numerical solution by mathematical means. The experiment can only verify the physical model in the case of accurate numerical solution. The accuracy of numerical solution must be verified by the mathematical methods.

In contrast, as demonstrated herein, the software developed at FPRI for fluid mechanical design of the Potential Flow turbine has the following advantages:

(i) It is based on correct formulation of the design problem.

(ii) It directly leads to the optimal solution.

(iii) It consists of highly accurate programs from the computational point of view. The accuracy of the programs has been checked by mathematical means.

Software Package for Fluid Mechanical Design

In the case of the Potential Flow turbine [1] a correct formulation of the design problem can be done easily since in this turbine the flow at the entrance to the runner is axisymmetric and, therefore, the relative flow at the entrance to the runner is steady and also axisymmetric.

The design problem for the Potential Flow turbine is to find the following, for given values of turbine head, H, flow rate, Q, and rotational speed, N:

(i) The geometrical shape of the wicket gate providing a flow at the entrance to the runner with constant value of the whirl, $(V_u R)_i$.

(ii) The geometrical shape of the runner blade providing given values H and Q at the specified N for the flow delivered to the runner trough the wicket gates.

The value of $(V_u R)_i$ is accepted to be equal to the change of the whirl between the entrance to the runner and it's exit, $\Delta(V_u R)$, and, therefore, the whirl at the runner exit to be equal to zero, $(V_u R)_e = (V_u R)_i - \Delta(V_u R) = 0$. The change of the whirl is defined by Euler equation:

$$\Delta(V_u R) = \frac{g\eta H}{\omega} \tag{1}$$

where:

g is acceleration due to gravity.
η is the efficiency of the turbine.
$\omega = \pi N/30$ is the angular velocity of turbine.

The Software Package for Fluid Mechanical Design of the Potential Flow turbine is based on quasi-three-dimensional approach. According to this approach the stream surfaces of the axisymmetric flow are determined and the cascades of profiles of the wicket gates and of the runner blades are designed in the laminae of variable thickness formed by adjacent stream surfaces of the axisymmetric flow. According to this approach the flow in each axisymmetric curvilinear lamina of variable thickness is assumed to be independent from the flow in other laminae. This assumption was theoretically justified for design of axial-flow runners by L. Simonov in 1941 [6], and supported by experimental data [2, 3]. The assumption was also theoretically justified for mixed-flow runners at FPRI in 1992 within the scope of contract with EPRI (to be published in 1996).

The computation of the axisymmetric flow is produced by the highly accurate program AXIFLOG based on finite element technique. The program AXIFLOG is described in the following section.

The Potential Flow wicket gates can not be designed using the method of singularities, because their geometrical shape must satisfy two geometrical constraints in order to have the capability to close the turbine water passages. The entrance edge of the Potential Flow wicket gate must be formed as a cylindrical surface parallel to the turbine axis and the trailing edge must pass through a straight segment also parallel to the turbine axis [1]. The wicket gate is designed in two steps. In the beginning it is designed using approximate assumption, that the angle, α, of the wicket gate profile trailing element with circumferential direction can be computed along the wicket gate span by following formula:

$$\alpha = arctan\left[\frac{(V_r R)_i}{(V_u R)_i}\right] \tag{2}$$

where: $(V_r R)_i$ was computed by AXIFLOG.

The second step in design of the Potential Flow wicket gate is based on the application of the subroutine CASFLOW for the analysis of the flow in the cascade with specified geometry using the the method of integral equations (see the section titled "Subroutine for Computation of the Flow in the Cascade"). The desirable distribution of the whirl along the wicket gate trailing edge, $(V_u R)_i = \Delta(V_u R)$, is achieved by an iterative process consisting of varying the angle α with subsequent computation of $(V_u R)_i$ by subroutine CASFLOW. The flow around the Potential Flow wicket gate is not optimized from the point of view of efficiency and cavitation, as described by the above design procedure. However, it is easy to see that the optimization of this flow is not necessary in this case due to two factors. The value of velocity in the flow passing the radial wicket gates is small and variation in the distribution of velocity in this flow can not significantly affect the turbine efficiency. There is no cavitation in the flow passing the wicket gates.

The runner blades are designed by the program INNA-2, generalized for the case of the turbine with mixed-flow runner (see the section titled "Program for Design of the Runner Blades"). The program INNA-2 is based on the method of singularities and allows optimization from the point of view of the cavitation coefficient, σ, and from point of the profile losses. This optimization is based on variation of the solidity of the cascades of the profiles forming the blades, the loading of the profiles (attached vorticity distribution) and the profile thickness distribution.

The software package also includes the programs for determination of the flow distribution and the loads in the already designed potential flow wicket gates and the runner blades for off design conditions. These programs are based on the application of the subroutine CASFLOW.

Program for Computation of Axisymmetric Flow

The program for computation of axisymmetric flow in the turbine water passages, AXIFLOG, is based on finite element technique. The accuracy of computations of this program was significantly improved by using the curvilinear orthogonal coordinate system to define the geometry of the finite elements. This permitted representation of the boundary of the water passages by sides of finite elements with a very high degree of accuracy . The accepted curvilinear orthogonal geometry of finite elements also allowed application of splining technique to the highly accurate semianalytical computation of Gallerkin integrals. In the finite element programs like ANTONY, with arbitrary definition of finite element geometry, the sides of the finite elements form a boundary of the polygonal shape. The Gallerkin integrals in these programs are computed inaccurately using the Gaussian method.

The problem of determination of axisymmetric flow in AXIFLOG is formulated in terms of the Stokes stream function, Ψ, where:

$$\frac{\partial\left(\dfrac{H_q}{RH_p}\dfrac{\partial\Psi}{\partial p}\right)}{\partial p}+\frac{\partial\left(\dfrac{H_p}{RH_q}\dfrac{\partial\Psi}{\partial q}\right)}{\partial q}=0 \tag{3}$$

where:

p and q are curvilinear orthogonal coordinates.
H_p and H_q are Lamé coefficients.
R is the radius from the turbine axis.

After determination of Ψ, velocity components are obtained numerically (using splining technique) according to the formulae:

$$V_p=-\frac{1}{H_qR}\frac{\partial\Psi}{\partial q},\qquad V_q=\frac{1}{H_pR}\frac{\partial\Psi}{\partial p} \tag{4}$$

The results of this finite element procedure were compared to an analytic solution in a simple situation permitting the latter [7]. The highest relative error in velocity was $< 0.01\%$. In order to obtain accuracy information in practice, we have built internal consistency checks into the program. These include:

(i) The constancy with respect to q of the difference in potential Φ_m from entrance to exit along along p-axis.

(ii) The constancy with respect to p of the flow rate Q_f along along q-axes from the entrance to the exit .

As has been proven by the numerous practical computations, the maximum relative deviation of Φ_m from average and the relative error in Q_f are $< 0.5\%$. One of these practical computations was for the Francis turbine at Butt Valley powerplant owned by Pacific Gas and Electric Company [1]. The plot of streamlines of axisymmetric flow for Butt Valley Francis as computed by AXIFLOG is presented in Figure 1. For complete mathematical description of the program AXIFLOG see [8].

Figure 1. Streamlines of Axisymmetric Flow for Butt Valley Francis

Program for Design of the Runner Blades

The program for design of the runner blades, INNA-2, is the generalization of the program INNA [2] for the case of the mixed-flow runner. The program INNA was developed by the author of this paper in 1982–1985, while at Allis Chalmers, for the design of axial-flow runner blades using the method of singularities. The program INNA-2 is based on the quasi-three-dimensional approach. This program designs the rotating cascades of profiles forming the runner blades. Each cascade is located in the lamina of variable thickness between two adjacent stream surfaces of the axisymmetric flow. The most important features of the program INNA-2 are:

(i) The flow in the cascade is mapped onto the straight cascade located in the flat lamina with variation of thickness in the direction perpendicular to the cascade using the the following formulae for conformal transformation:

$$\Xi = \Phi R_0 \tag{5}$$

$$H = \int_0^{\ell_s} \frac{R d\ell_s}{R_0} \tag{6}$$

$$(V_c)_\xi = \frac{R}{R_0} V_u \tag{7}$$

$$(V_c)_\eta = \frac{R}{R_0} V_m \tag{8}$$

where:

Ξ and H are the Cartesian coordinates in the plane domain (the axis $O\Xi$ goes along the cascade).
R and Φ are the cylindrical coordinates for the point along the stream surface.
ℓ_s is the length along a streamline of the stream surface.
R_0 is the value of R for the point at a streamline of the stream surface with $\ell_s = 0$.
$(V_c)_\xi$ and $(V_c)_\eta$ are absolute velocity components in the plane domain.

(ii) The method of singularities is used for the design of the profiles of the straight cascades on the plane domain $O\Xi H$ [9]. The relative variable thickness of the flat layer, $h = h(\eta)$, was approximated by parabolic function: $h = (1 + \alpha\eta)^2$. The analytical formulae for the velocity fields generated by single source/sink and vortex for the flat parabolic layer were developed by author of this paper and Dr. Y.V.N. Rao in 1965–1967 [10, 11]. The formulae for velocity fields generated by singularities (vortices and sources/sinks) distributed along the line of singularities located inside of each profile were obtained by integration using the formulae for single source/sink and vortex.

In the cases where the analytical integration was not possible, the highly accurate integration using splining technique was used.

(iii) The geometrical shape of the conformal mapping of profile of the straight cascade and the velocity distribution around it are computed on the domain $O\Xi H$. The geometrical shape of the profile forming the runner blade and the velocity distribution around it are obtained by inverse conformal mapping from the domain of straight cascade onto the domain of cascade rotating in the lamina between stream surfaces of axisymmetric flow using the formulae (5)–(8).

Subroutine for Computation of the Flow in the Cascade

The subroutine CASFLOW for computation of the flow in the rotating or stationary cascade located in the lamina of variable thickness between two adjacent stream surfaces of the axisymmetric flow is based on the method of integral equations. The most important features of the solution are:

(i) As in the program INNA-2, the flow in the cascade is mapped onto the straight cascade located in the flat lamina with variation of thickness in the direction perpendicular to the cascade using the formulae (5)–(8). The method of integral equations is applied to this straight cascade. The flow through the cascade of profiles located in the lamina between two adjacent stream surfaces is obtained applying the formulae (5)–(8) to the flow in the straight cascade.

(ii) The determination of the flow through the straight cascade is based on the Douglas Neuman approach [12] to the method of integral equations. The Douglas Neuman method was generalized for the case when the straight cascade was a result of conformal mapping of the cascade rotating in the lamina of variable thickness between two adjacent stream surfaces of the axisymmetric flow and the computational accuracy of this method was significantly improved. The first improvement of accuracy was achieved by considering the unknown sources/sinks distribution along the profile, $q = q(\ell)$, as the polygonal function with nodal points at the vertices of the segments representing the profile boundary (in Douglas Neuman method $q = q(\ell)$ was represented by a step function with a constant value of q along each segment). The second improvement of accuracy was achieved by improving the formula for velocity at vertices generated by two adjacent segments by considering one smooth curvilinear segment instead of two straight segments. The generalization of the Douglas Neuman method for the case of variable thickness required new formulae for velocities generated by polygonal distribution of $q = q(\ell)$ with unknown values of q_i at nodal points. These formulae were obtained by integration using the same analytical formulae for the velocity fields generated by single source/sink and vortex for the the flat parabolic layer, as in the case of the program INNA-2. According to the Douglas Neuman method for stationary cascade, three linearly independent flows are found: the flow with

$V_\eta = 1$ at $\eta = \infty$, the flow with $V_\xi = 1$ at $\eta = \infty$, and the flow generated by $\gamma = 1$ distributed along all profiles. The flow in the cascade is a result of linear combination of these flows satisfying the boundary conditions at $\eta = \infty$. The generalization of the Douglas Neuman method for rotating cascade required the fourth linearly independent solution for the flow V_{rt} satisfying the boundary condition at the profiles:

$$(V_{rt})_n = \frac{\omega R^2}{R_0} \cos\beta \tag{9}$$

where: β is the angle between normal to the profile and axis $O\Xi$.

Figure 2. Flow around Butt Valley Runner Blade Profile Computed by INNA-2 and CASFLOW

The subroutine CASFLOW was verified using the program INNA-2. This verification showed that the differences in computations of velocity distribution around the profiles are within 1.0%. Figure 2. presents the flow distributions computed by INNA-2 and by CASFLOW around Butt Valley runner blade profile designed by INNA-2.

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Key Words

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